

Latest Developments in CFDEM®coupling and LIGGGHTS®

„Dedicated to open source high performance scientific computing in fluid mechanics and particle science“

Dr. Christoph Kloss, Dr. Christoph Goniva

office@dcs-computing.com

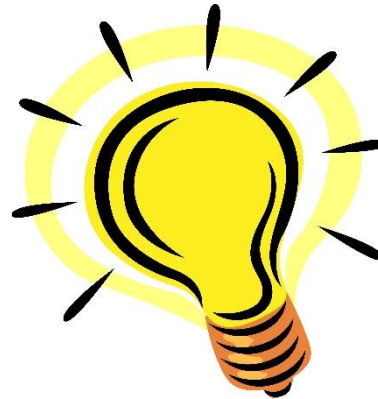
www.cfdem.com

DCS Computing GmbH, Linz

Outline

- Share the **latest news** about **CFDEM®project** and **recent developments**

- Share and exchange modelling **ideas**



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CFDEM®
COUPLING

CFDEM®
PROJECT

Intro

CFDEM® project latest news

Company Focus

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Modelling Flows of Particles, Gases and Liquids: CFD-DEM technology

Particles & flow processes are everywhere

Sugar, sand, ores, tablets, chemicals, biomass, detergents, plastics, crops, fruits need to be harvested, produced, processed, transported, stored.

DCS Computing is specialized in modelling and engineering solutions for these particle and fluid flow processes.



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Business Units

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I
RESEARCH



II
SIMULATION
ENGINES



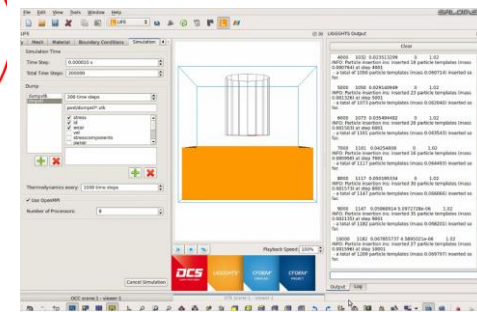
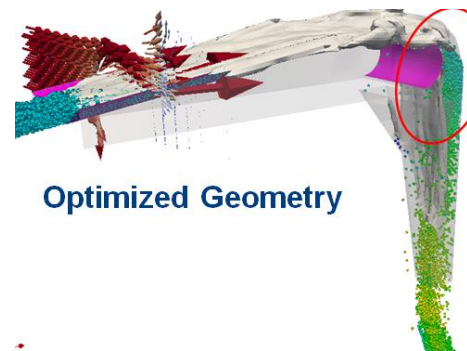
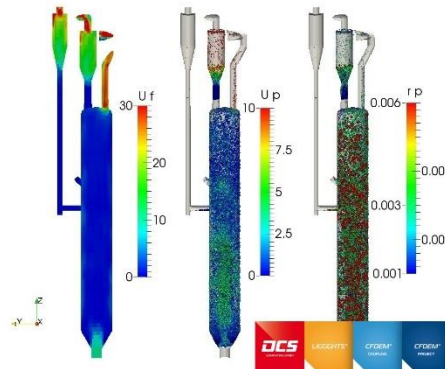
III
CAE
CONSULTANCY



IV
CAE
Software

$$\Pi_2 = \frac{k_n}{R_i \cdot \rho_p \cdot v_0^2}$$

$$\Pi_3 = \frac{c_n}{R_i^2 \cdot \rho_p \cdot v_0}$$



DCS Computing covers the whole range from physics, code design of simulators, simulation model development, software/workflow development, support and training to actual engineering applications

All levels of creation of value with simulation technology are covered

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History & Timeline of DCS

- Founded 2012, Headcount 9 (July 2015)
- CAGR 89% (2012 turnover vs 2015 estimated turnover)
- DCS is funded by project cash flow, which is fully driven by the customers' needs and demands.
- DCS activities well balanced over business areas/industries



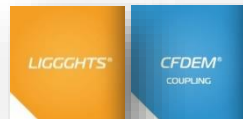
Impact: CFDEM®coupling workbench (FFG)



Fundament: Technology development with customers, 2 EU FP7 projects



Pre-Development outside DCS



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Why use DEM and CFD-DEM?

Over 70% of industrial processes involve particles **BUT**

- majority of particle handling/processing operations empirically designed
- Measurement and control is difficult and costly.

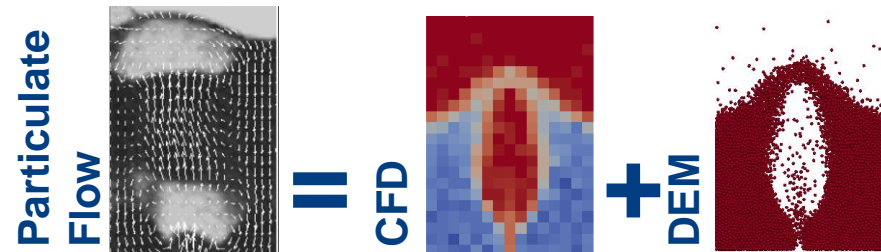
(CFD-)DEM is used by **engineers** worldwide to increase profits by:

- **Reducing** need for **physical prototypes**
- **Troubleshooting** operational problems
- Designing more **efficient processes** by providing hard-to-measure information on bulk and particle-scale behavior
- Saving expensive **trial and error**



All processes include fluid-particle interaction

- **neglecting that often leads to errors!**
- Many processes inherently based on fluid-particle interaction
- Measurement is difficult and costly



Theoretical background – coarse grained CFD-DEM:

Navier-Stokes equations for the fluid in presence of a granular phase

$$\frac{\partial \alpha_f \rho_f}{\partial t} + \nabla \cdot (\alpha_f \rho_f \mathbf{u}_f) = 0$$

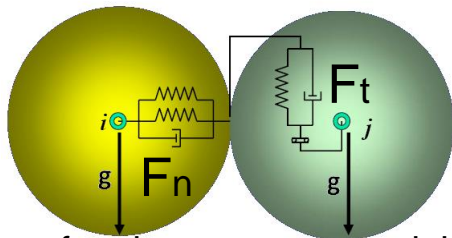
$$\frac{\partial (\alpha_f \rho_f \mathbf{u}_f)}{\partial t} + \nabla \cdot (\alpha_f \rho_f \mathbf{u}_f \mathbf{u}_f) = -\alpha_f \nabla p + \nabla \cdot (\alpha_f \boldsymbol{\tau}) + \alpha_f \rho_f \mathbf{g} - \mathbf{K}_{fs} (\mathbf{u}_f - \mathbf{u}_s)$$

Lagrangian Particle Trajectory for **Parcels**

$$\frac{\partial^2 \mathbf{x}_p}{\partial t^2} = \frac{\mathbf{F}_n}{m_p} + \frac{\mathbf{F}_t}{m_p} + \mathbf{g} + \frac{\beta}{\rho_p \alpha_p} (\mathbf{u}_f - \mathbf{u}_p) - \frac{1}{\rho_p} \nabla p$$

Scaling laws from dimensional analysis

$$\Pi_1 = l, \Pi_2 = \frac{k_n}{R_i \cdot \rho_p \cdot v_0^2}, \Pi_3 = \frac{c_n}{R_i^2 \cdot \rho_p \cdot v_0}$$



soft-sphere contact model:
linear spring-dashpot

- l : size ratio of colliding particles, k_n : stiffness, R : radius, ρ : density, v_0 : reference velocity
- **scaling stiffness**
- **scaling of particle drag**
- **Equations converge to particle equation for parcel = particle**

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Modelling needs—Fluidized Bed

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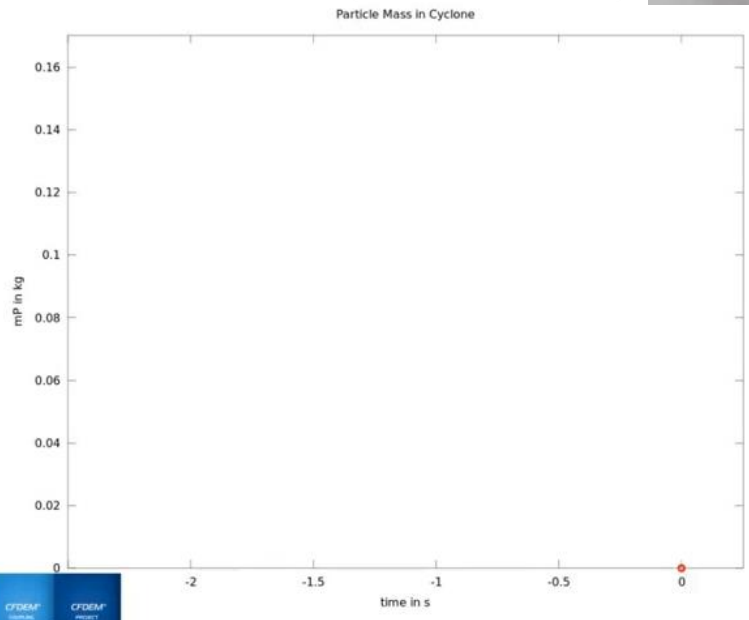
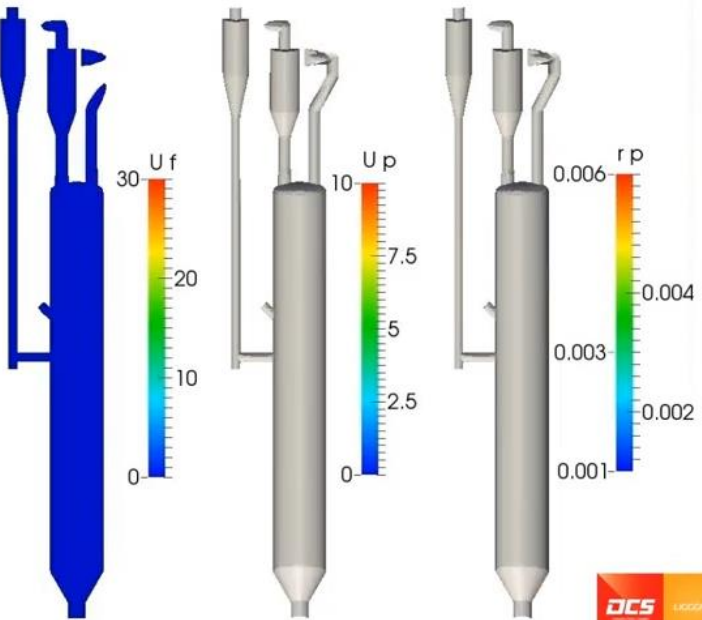
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Fluidized Bed Processing

Process with high interphase exchange rates
Applications in most process industries,
including processes w/ high CO₂ production

- Optimize heat/mass transfer
- Where do the fines go?, Is there segregation?
- Capture CO₂ in the process



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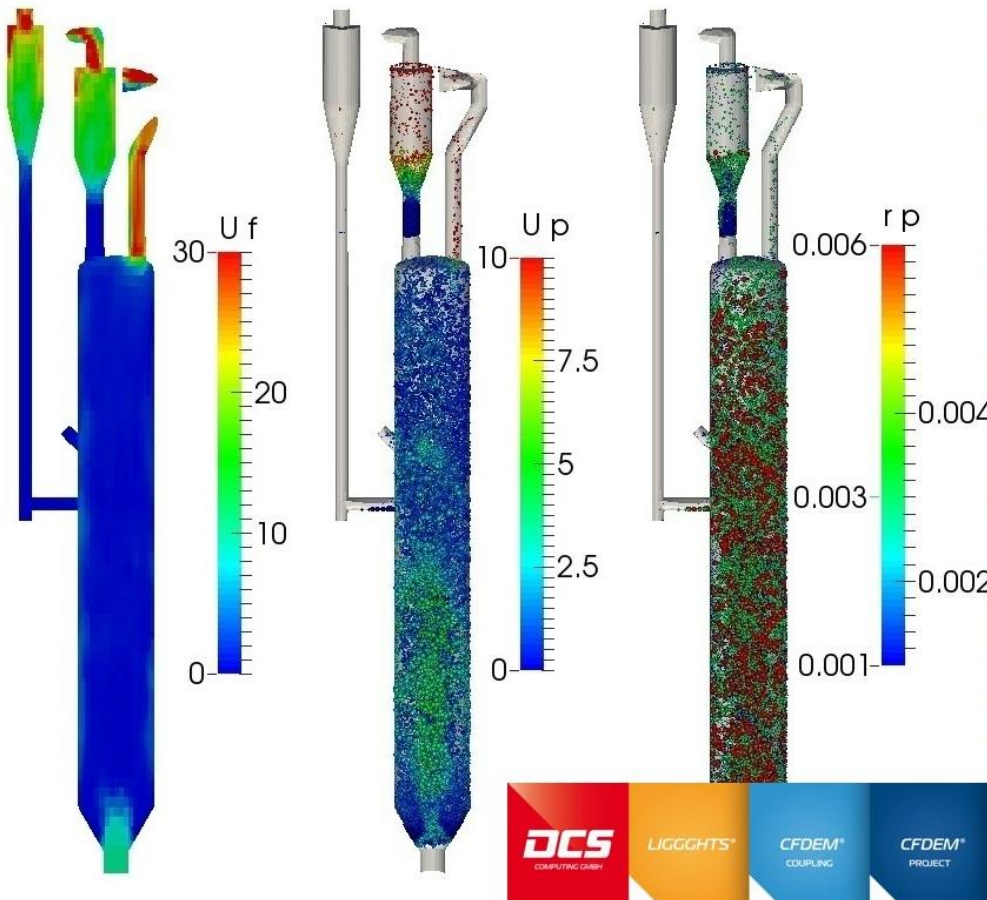
Fluidized Bed Modelling

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What needs to be modelled?

- **Particle dynamics** (solid phase)
- **Fluid dynamics** (gas phase)
- **Inter-phase transport processes**
particle-fluid momentum transfer,
particle-fluid heat & mass transfer
- **Intra-phase transport processes**
Intra-Particle heat transfer
Intra-Particle chemical reactions

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Eco-system is growing!

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What needs to be modelled?

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Eco-system is growing!

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parScale



**New! First release was a new year gift
(31 Dec 2014), next one to follow soon!**

For download: www.cfdem.com



- **Intra-phase transport processes**
Intra-Particle heat transfer
Intra-Particle chemical reactions

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Mastermind

(right hand side)



New! First release was a new year gift
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- 
- **Intra-phase transport processes**
 - Intra-Particle heat transfer
 - Intra-Particle chemical reactions

PaScal Library for Particle-Based Modelling of Chemical Reactions

- The particle scale simulation tool with interface capabilities to LIGGGHTS, OpenFOAM, FLUENT, NEPTUNE_CFD, as well as to the reaction modelling tool REMARC.

various particle-scale models to be able to solve complex reaction-diffusion problems, and will have subroutines to model drying or devolatilization processes

The logo for PaScale is rendered in a stylized, rounded font. The letters 'pa' are yellow, 'S' is orange, and 'cale' is red. The letters have a slight 3D effect with a dark shadow on the right side.

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Software Testing

- The test harness is a development tool to **build binaries, run test cases** and **check for consistency** with previous versions.
- It can perform this jobs on the **local machine** or an **external cluster**.
- The **web-based frontend** allows to check the results of all test cases for several builds at once and from your working computer
- The test harness was originally developed for the development of LIGGGHTS® and was extended to cover also CFDEM®coupling and ParScale.
- CI server is used to **check automatically for new commits** to the projects and **start (if required) the test harness**.
- If something is broken, **notification email** to the user is sent.
- In active use for development

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Testing: LIGGGHTS®



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⚙️ liggghts-src-unstable-3.0.x ▶

LIGGGHTS

🔗 Show version consistency compared to liggghts-src-unstable-3.0.2

📁 examples



✓ 175

✗ 2

⚠️ 15

📁 local



✓ 3

✗ 0

⚠️ 145

⚙️ pascal-master-0.x ▶

PaScaL

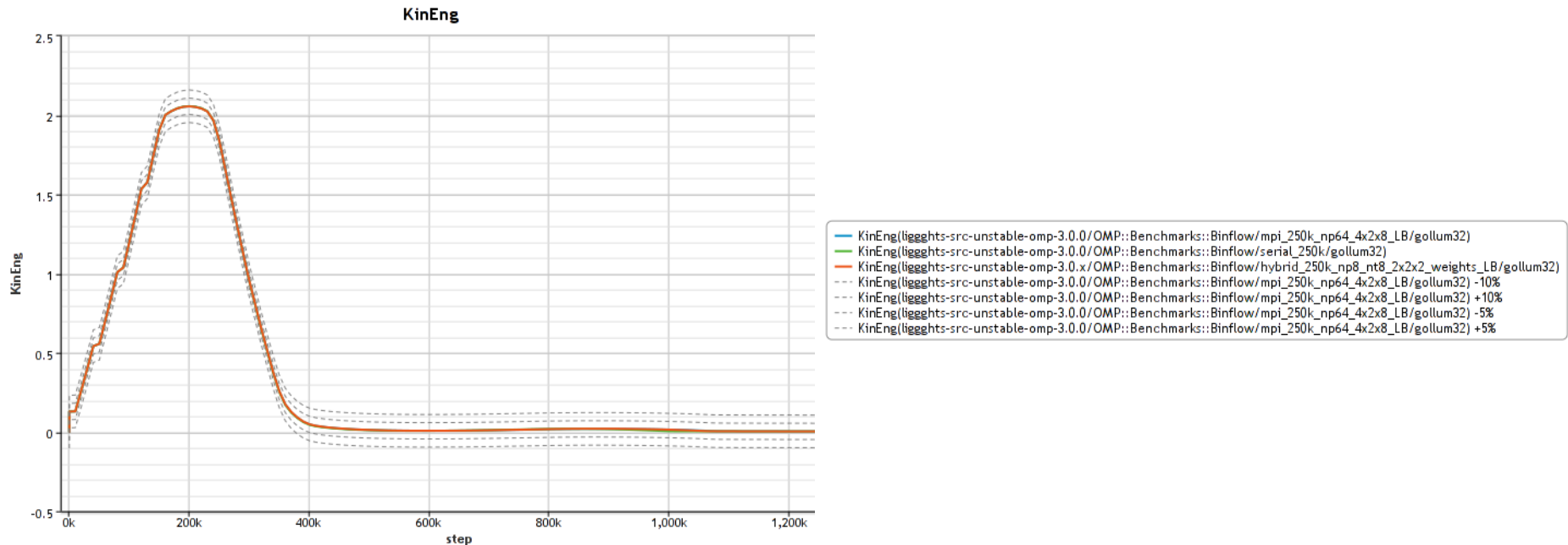
📁 pascal-examples



✓ 3

✗ 1

⚠️ 0



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Testing: CFDEM®coupling



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CFDEMproject Test Harness

Overview

Comparisons

Search



Status

Andreas Aigner

Comparison

+ Add all

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- CFDEMcoupling-DCS-master-liggghts-src-unstable-OF-2.4.x/CFDEMcoupling-DCS-master-liggghts-src-unstable-OF-2.4.x/DCS-examples::tutorials::cfdemSolverPiso::ErgunTestMPI/cfdemrun/glados

Overview

Logfile

Input Script

Files

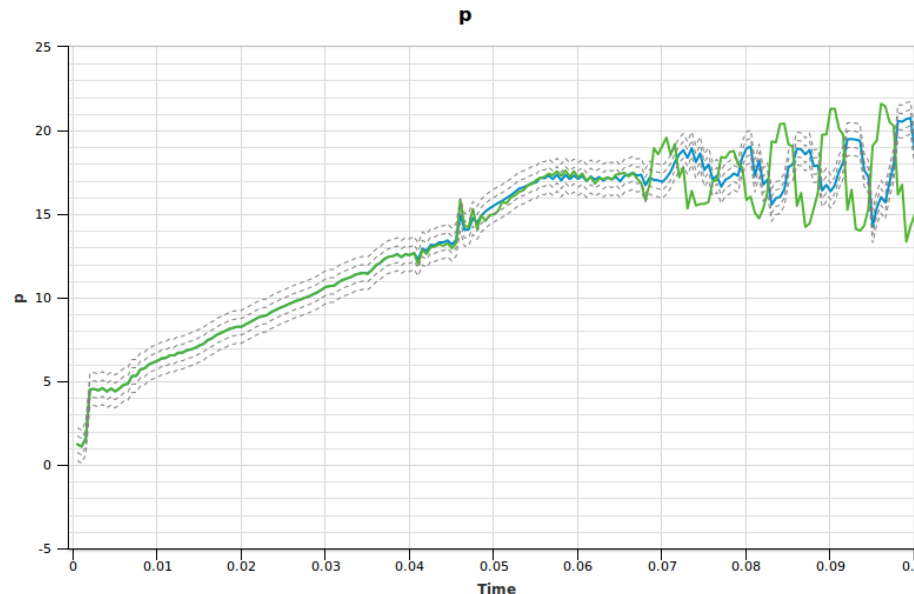
Fix Timing

Plots

Probes

probes: (3e-05, 0, 0.0001)

p(3e-05, 0, 0.0001)



coupling-DCS-master-2.8.x-liggghts-src-unstable-3.2.x-OF-2.3.x/DCS-examples::tutorials::cfdemSolverPiso::ErgunTestMPI/cfdemrun/g
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coupling-DCS-master-2.8.x-liggghts-src-unstable-3.2.x-OF-2.3.x/DCS-examples::tutorials::cfdemSolverPiso::ErgunTestMPI/cfdemrun/g

Highcharts.com

U/(3e-05, 0, 0.0001)

CFDEMcoupling-DCS-master-2.8.x-liggghts-src-unstable-3.2.x-OF-2.3.x

Consistency: None

Detail:

CFDEMcoupling-DCS-master-liggghts-src-unstable-OF-2.4.x

Consistency: 0.0

Detail:

- plot thermo_KinEng inconsistent: 0.000000
- plot thermo_rke inconsistent: 0.000000
- plot thermo_dragtota inconsistent: 0.000000

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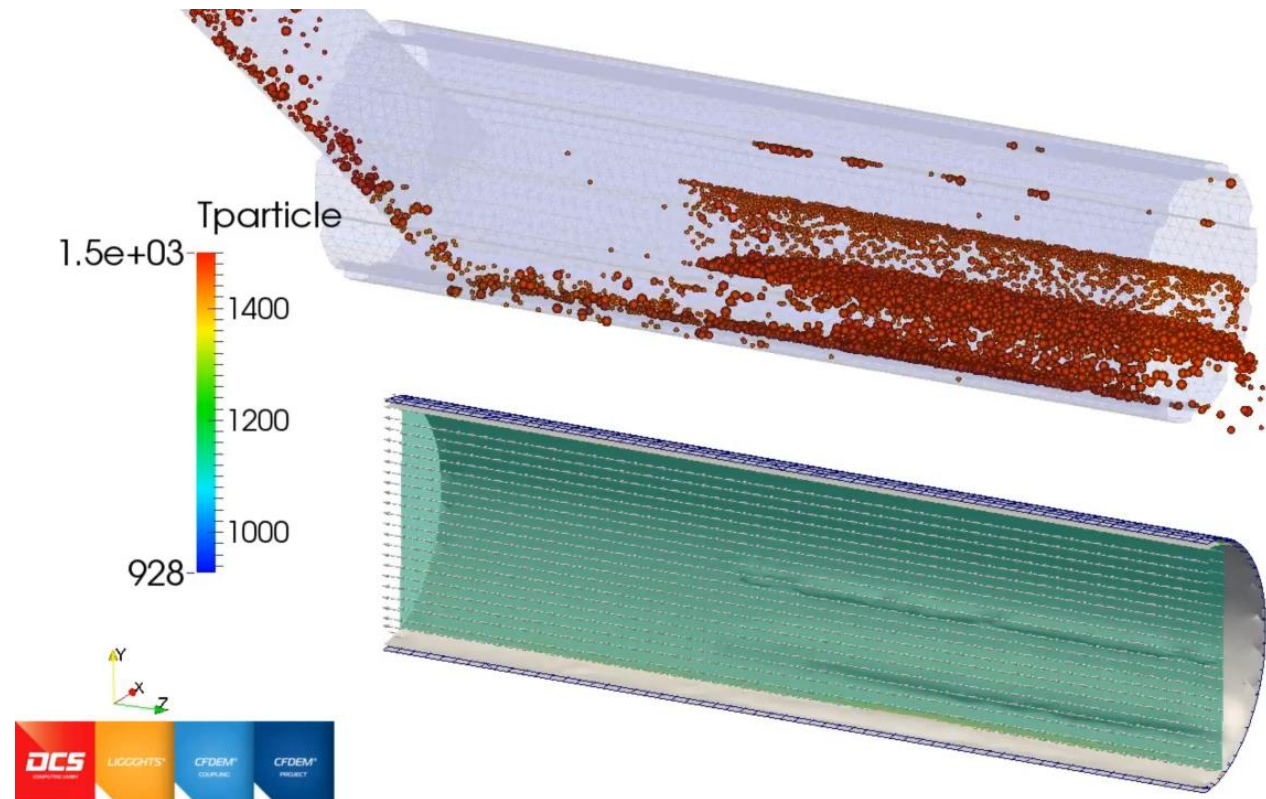
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Recent Developments on CFDEM® coupling

Generic Scalar Transport Model coupled CFD-DEM simulations

Generic scalar transport model class that can be hooked onto different solvers, where sub-models for scalar transport can be implemented (temp., species, etc)



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CFD-DEM domain decomposit.

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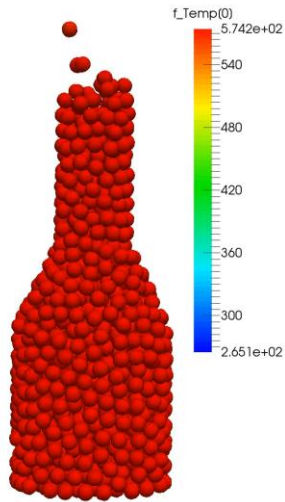
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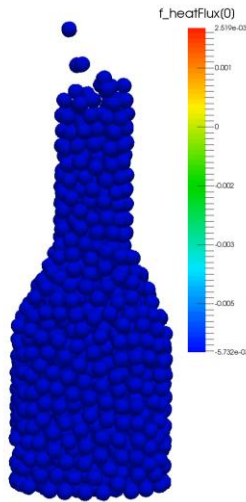
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Moving bed reactor with convective heat transfer

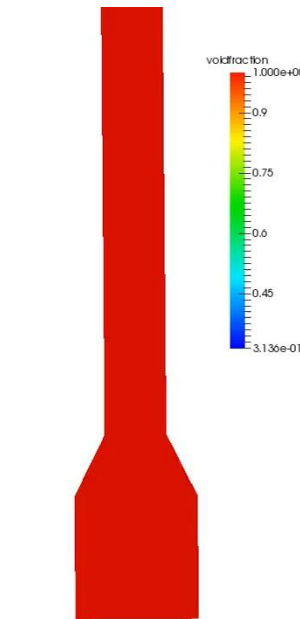
Time: 0.000000



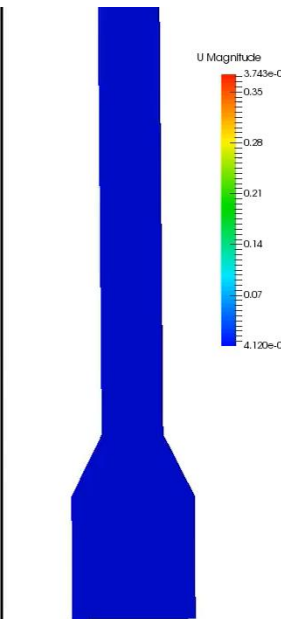
Particle Temperature



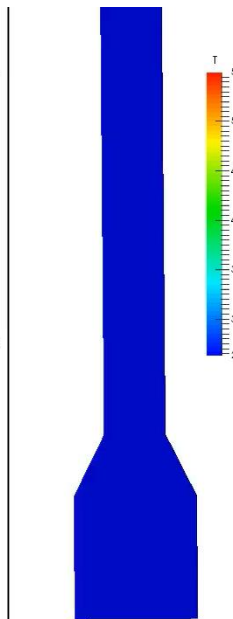
Particle Heatflux



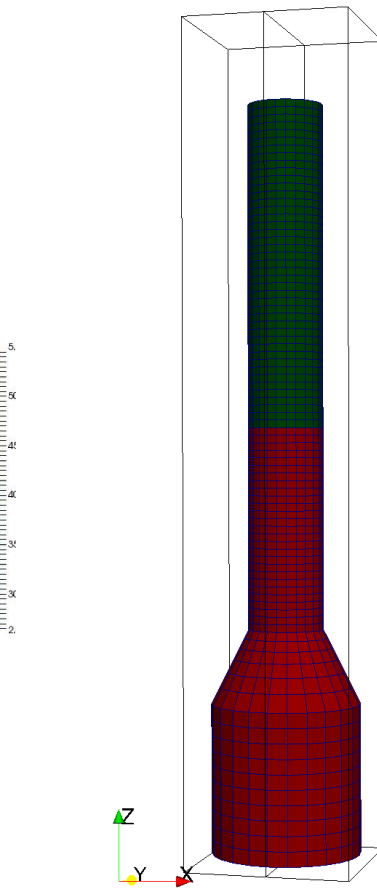
voidfraction



Ufluid

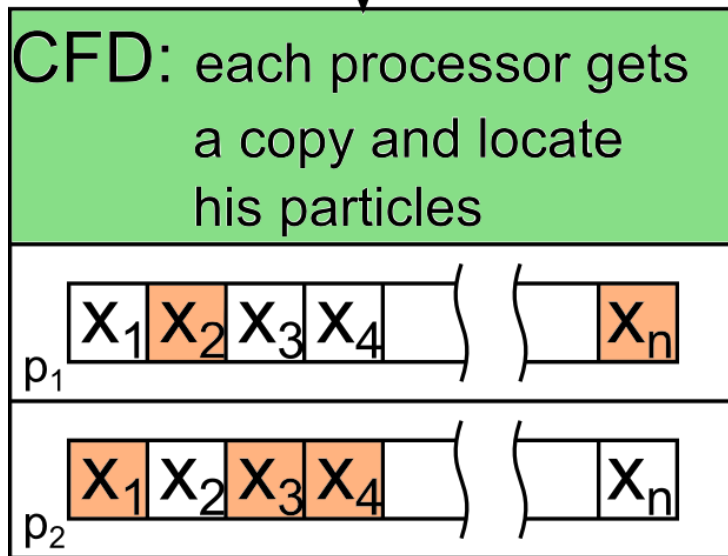
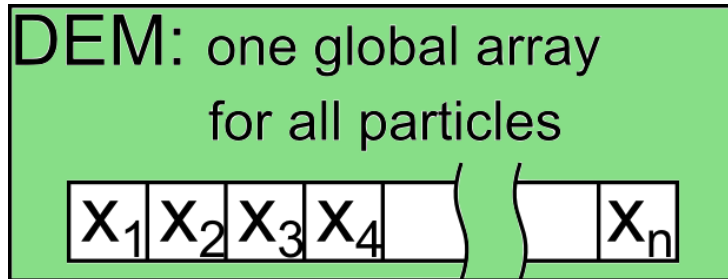


Tfluid

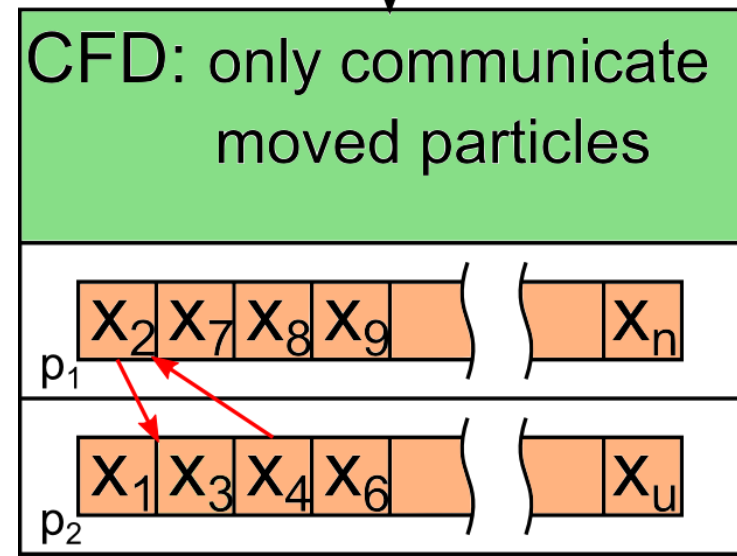
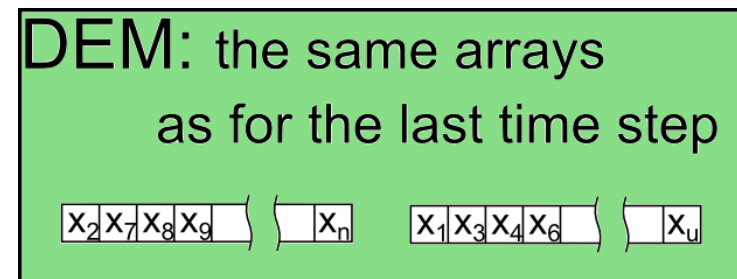


Red/Green: Decomp. CFD
Black Lines: Decom. DEM

All-To-All



Many-To-Many



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ManyToMany Communication

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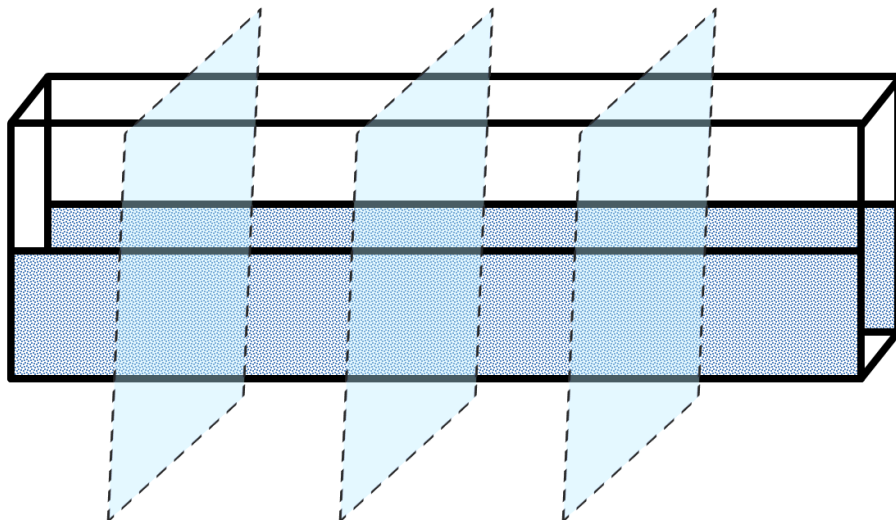
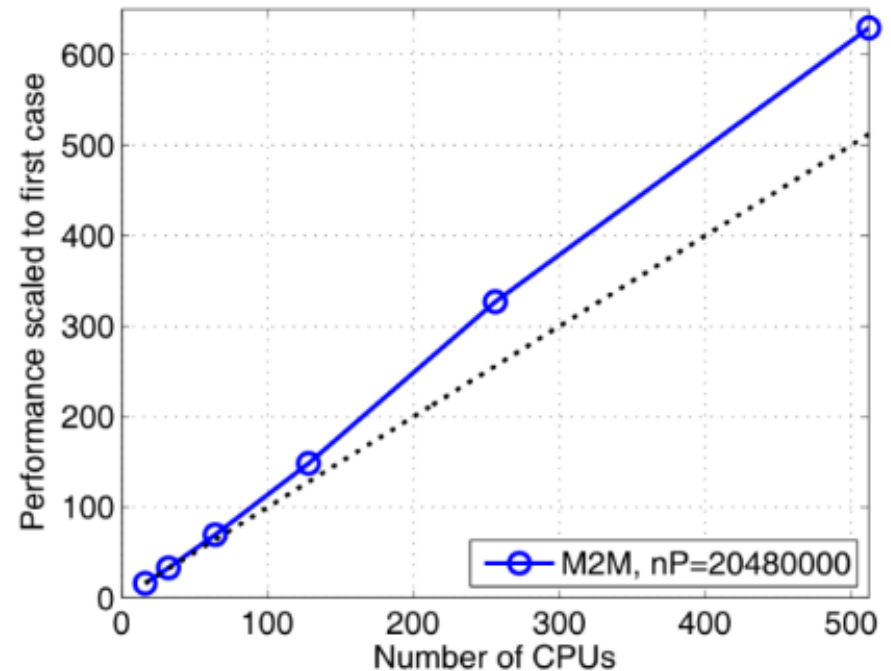
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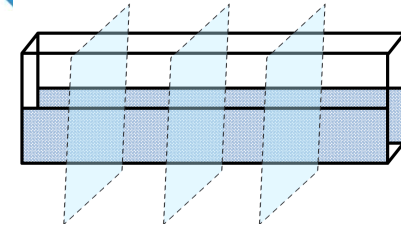
Test Case	Purpose	Machine used	Version of “many2many” used
Elongated Packed Bed #1	Show feasibility of the approach and its scalability	JKU cluster “MACH” (512 CPUs)	Very early feasibility version
Elongated Packed Bed #2	Show scalability of the final stable version and applicability across clusters	JKU cluster “LISE” (128 CPUs)	Stable version 1.0
Thermal Packed Bed	Compare results to physical lab-scale test-case where experimental data is available	Local workstation and JKU cluster “Gollum” (one blade with 32 cores)	Stable version 1.0

Elongated packed Bed #1

The system consists of a block 10.24 x 0.002 x 0.1 m / 10240 x 2 x 100 cells, filled with particles of $d_p = 0.3$ mm, and a total particle number of $n_p = 20.48e6$.

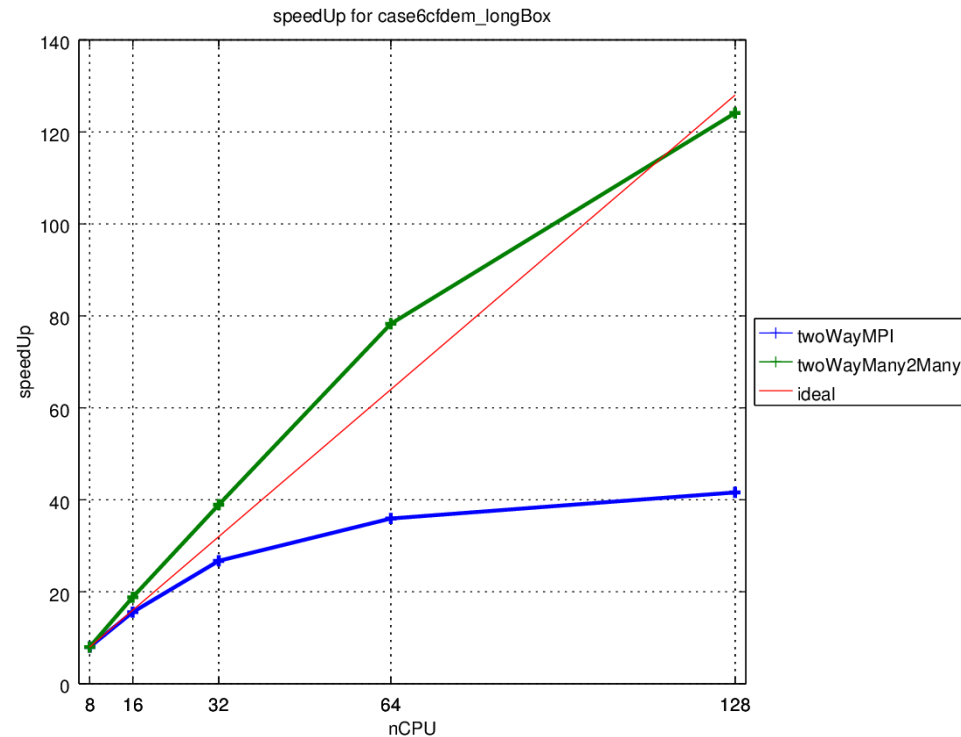
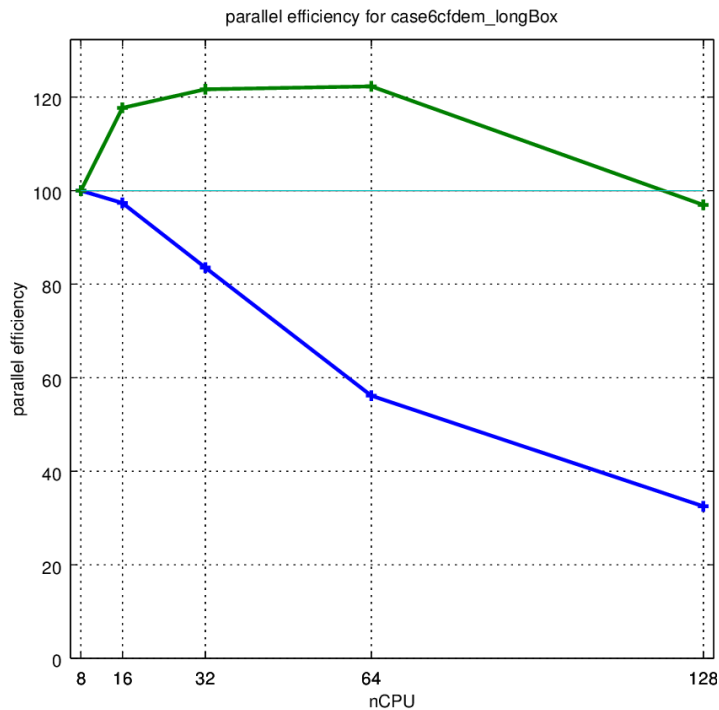
Parallel Scaling for:Global





Elongated packed Bed #2

The system consists of an elongated block of 64x1x1 m, is filled with 1,1 Mio particles (radius of 0.02m) and represents a packed bed which is seeing gas flow through the bottom walls with a velocity of 0.6077 ms/s, starts to bubble slightly



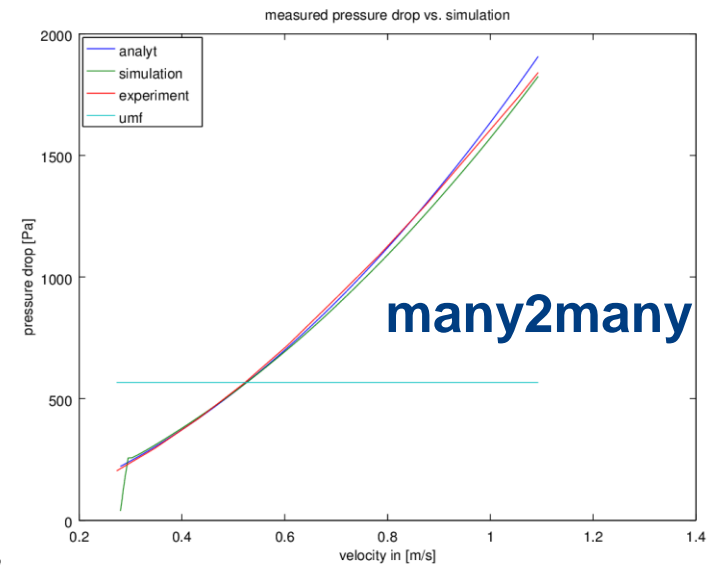
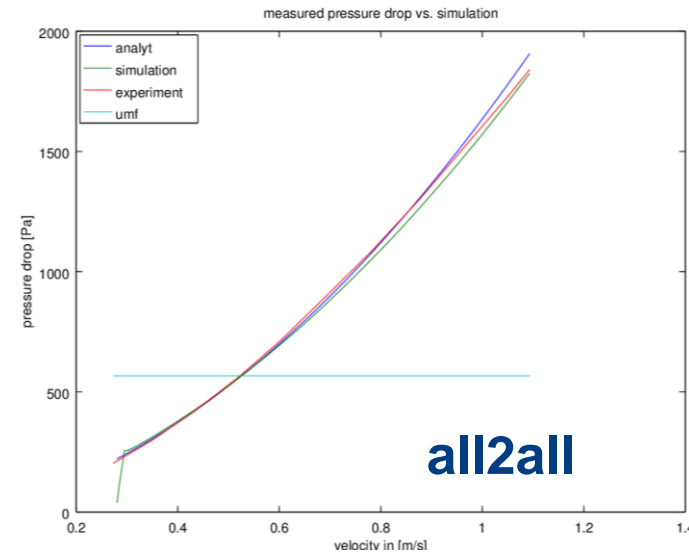
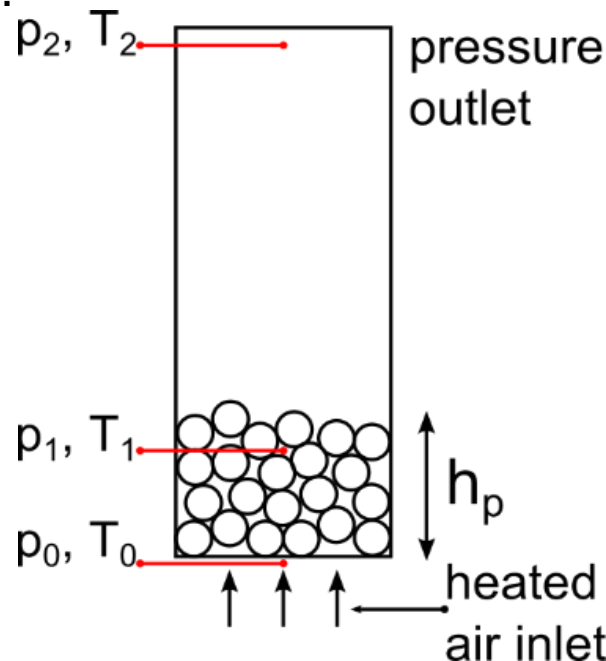
Thermal Packed Bed

Thermal Packed Bed

cylindrical fixed bed ($d_{cyl}=0.1\text{m}$, $h_p=0.1\text{m}$) with heated air inlet where experimental data (courtesy of JKU) was available. Particles are non-spherical poly-propylene-particles with Sauter mean diameter of 3 mm. Pressure and temperature are measured. The packed bed is fixed by a porous plate so it can not fluidize.

Conclusions:

- (i) the numerical method can capture the physics of the process, and
- (ii) the Many2Many and the All2All scheme deliver the same macroscopic result.



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3 Phase Melting CFD-DEM

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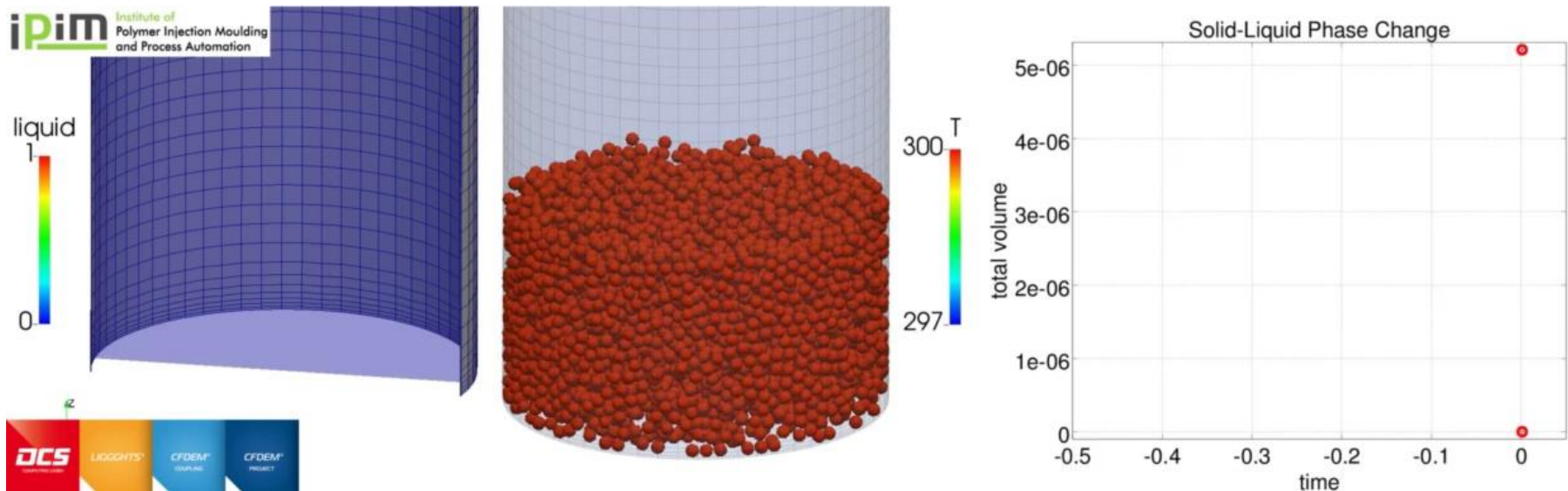
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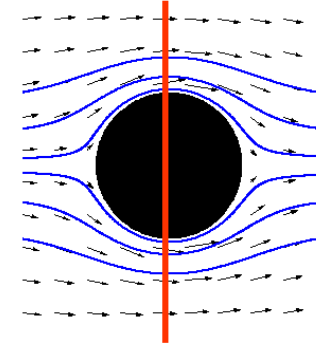
Result

- A model using VOF approach to capture solid-liquid phase change
- Energy equation for particle melting

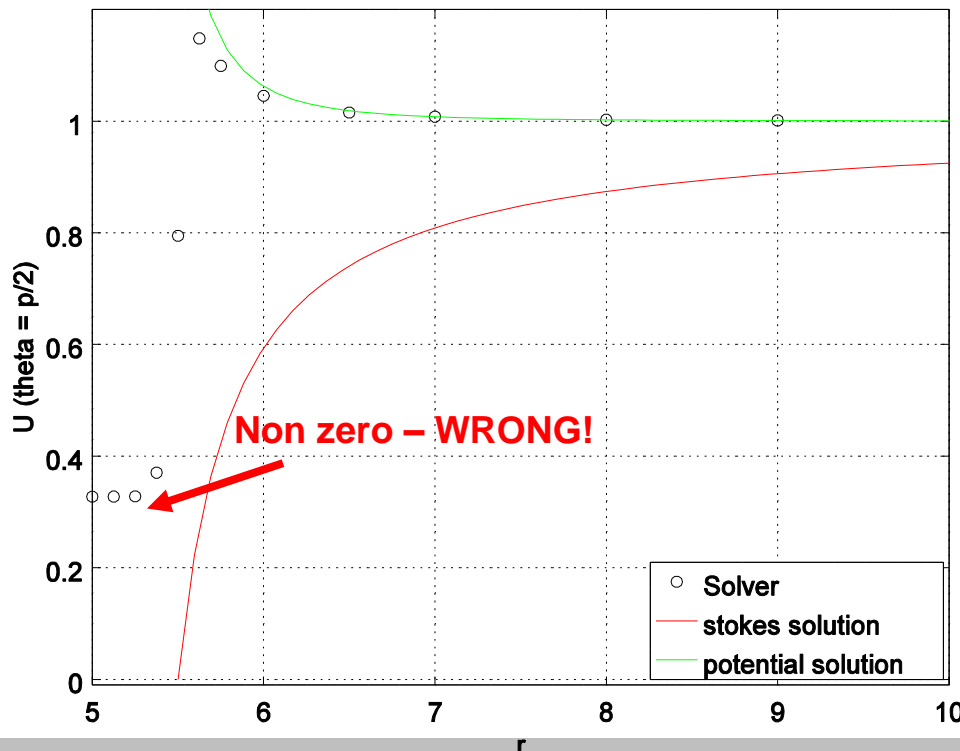


Improved Immersed Boundary Model

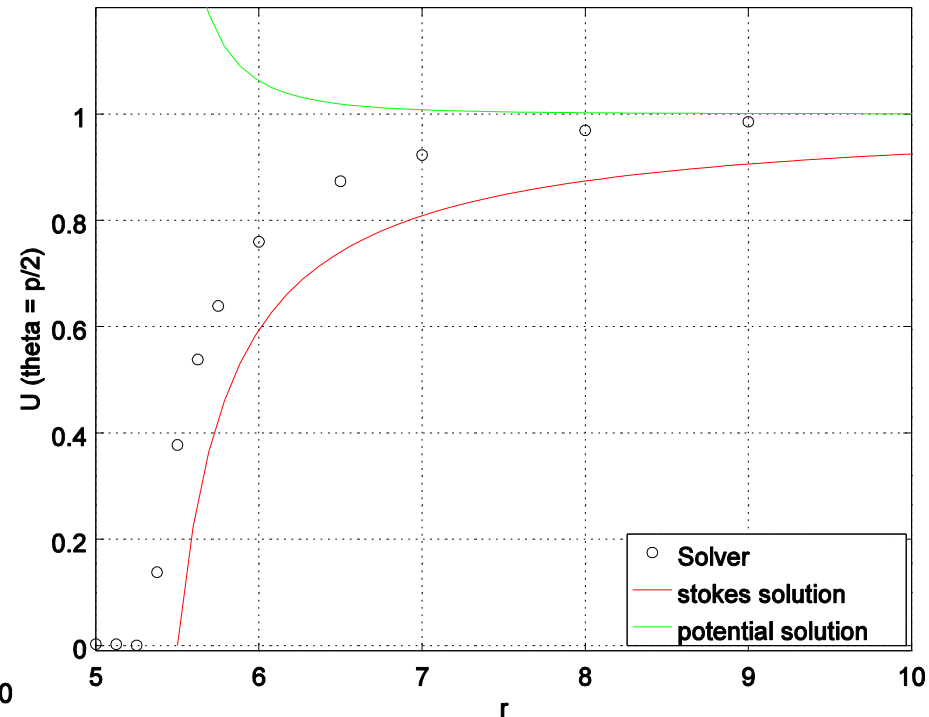
- Flow past sphere
- Improved solver (right) fulfils no-slip condition on sphere surface



Stokes Flow past a sphere



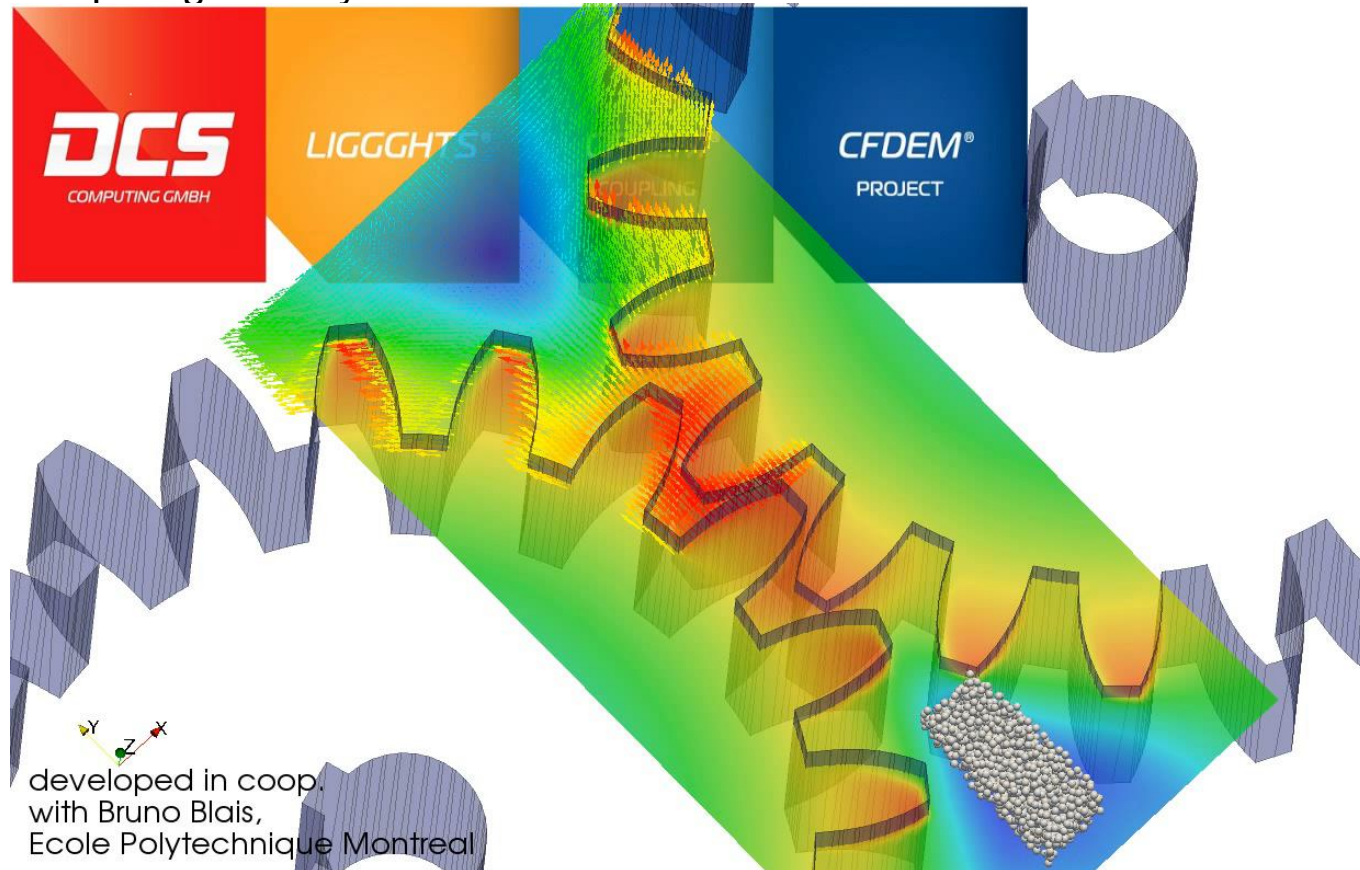
Stokes Flow past a sphere



Hybrid IB Models

Hybrid IB Model

- First version of a hybrid IB (for solid parts) with CFD-DEM (for particles) model.
- It allows complex geometry motion



Latest Dev. & Outlook

Spray Coating

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Physics to be covered

- Spray modelling
- Spray-particle interaction
- Liquid bridge forces
- Liquid transport btw. particles

Spray modelling

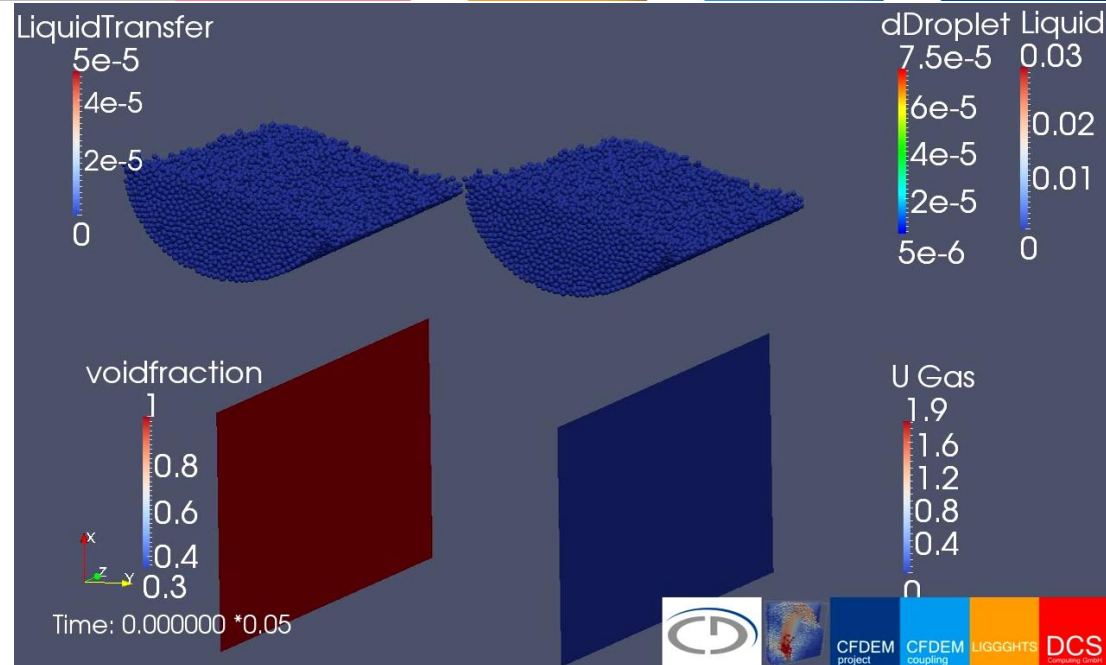
- Equation of Motion

$$m_D \frac{dv_D}{dt} = g(\rho_D - \rho_G)V_D + C_{d,D}A_D \frac{\rho_G(v_G - v_D)|v_G - v_D|}{2}$$

- Drag Law $C_{d,D} = C_{d,sphere}(1 + 2.632 y)$

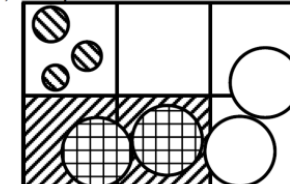
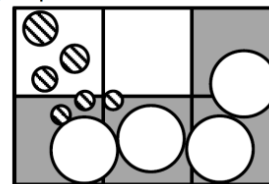
- Breakup Model (e.g. O'Rourke*)

$$\ddot{y} + \frac{5\mu_D}{\rho_D r^2} \dot{y} + \frac{8\sigma}{\rho_D r^3} y = \frac{2\rho_G v_{rel}^2}{3\rho_D r^2}$$



Spray-particle interaction

a) Liquid Source Detection: b) Droplet-Particle Transfer:



- DEM particle
- ⊗ droplet
- CFD cell
- voidfraction < 1
- ▨ liquid source field
- ⊕ particle gaining liquid

C. Goniva, J. Kerbl, S. Pirker, C. Kloss: Modelling Spray Particle Interaction by a Coupled CFD-DEM Method, Proc. Computational Modelling Conference 2013

Latest Dev. & Outlook

Spray Particle Collision Model

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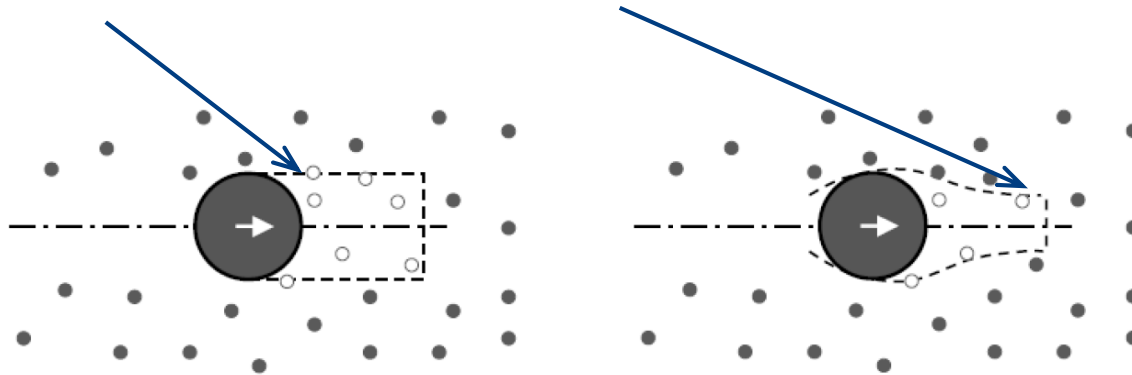
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Basic Idea:

Particles collect spray on their surface according to their collisional cylinder, which is corrected by Stokes Nr. dependent correlation:



From Goniva, PhD thesis.

Assumptions:

- Collisional regime is dominant.
- Particles are bigger than spray.
- Particle diameter is constant for whole domain.

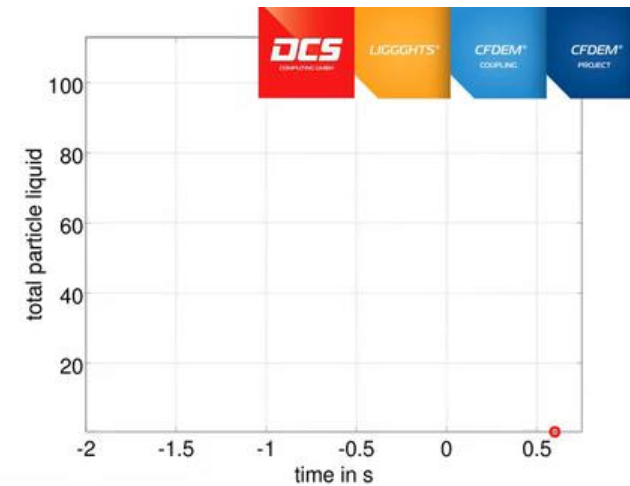
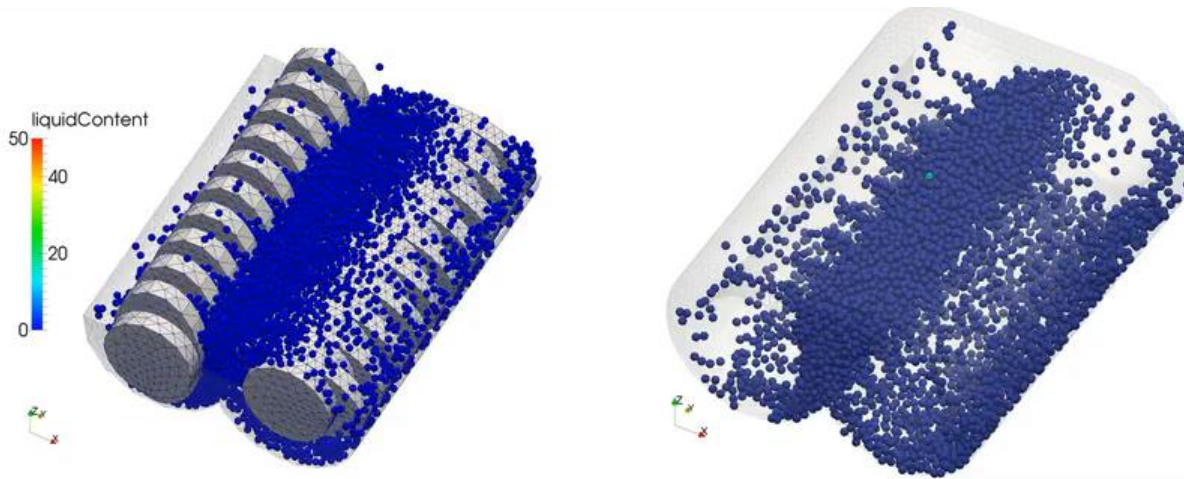
Literature:

GONIVA C., TUKOVIĆ Ž. and PIRKER S. (2009): "Simulation of offgas Scrubbing by a Combined Eulerian-Lagrangian Model", International Conference on CFD in the Minerals and Process Industries CSIRO, Melbourne, December 9-11.

HÄHNER F., (1994), "Inertial impaction of aerosol particles on Single and Multiple Spherical Targets", Chem. Eng. Technol., 17, 88-94.

Latest Dev. & Outlook

Spray Coating



Spray modelling

➤ Equation of Motion

$$m_D \frac{dv_D}{dt} = g(\rho_D - \rho_G)V_D + C_{d,D}A_D \frac{\rho_G(v_G - v_D)|v_G - v_D|}{2}$$

➤ Drag Law

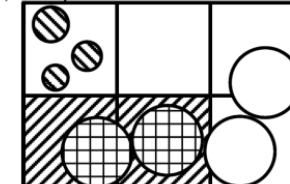
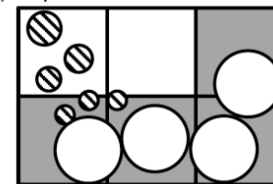
$$C_{d,D} = C_{d,sphere}(1 + 2.632 y)$$

➤ Breakup Model (e.g. O'Rourke*)

$$\ddot{y} + \frac{5\mu_D}{\rho_D r^2} \dot{y} + \frac{8\sigma}{\rho_D r^3} y = \frac{2\rho_G v_{rel}^2}{3\rho_D r^2}$$

Spray-particle interaction

a) Liquid Source Detection: b) Droplet-Particle Transfer:



- DEM particle
- ◐ droplet
- CFD cell
- voidfraction<1
- ▨ liquid source field
- ⊕ particle gaining liquid

C. Goniva, J. Kerbl, S. Pirker, C. Kloss: Modelling Spray Particle Interaction by a Coupled CFD-DEM Method, Proc. Computational Modelling Conference 2013

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Recent Developments on LIGGGHTS®

Major facelift version

- Revision of src:
2326 files changed,
git diff is 3,000,000 lines
- **Revision of documentation:**
lines changed:
1,383 additions 21,065 deletions
- Headers changed (see pic),
Copyright and License clarified
- Pure LAMMPS functionalities
removed completely from src, doc
- New file structure, new packages
- Fixed bugs in the build script
- Other improvements / bug-fixes:
<http://www.cfdem.com/node/42>

This is the



DEM simulation engine, released by
DCS Computing GmbH, Linz, Austria
<http://www.dcs-computing.com>, office@dcs-computing.com

LIGGGHTS® is part of CFDEM®project:
<http://www.liggghts.com> | <http://www.cfdem.com>

Core developer and main author:
Christoph Kloss, christoph.kloss@dcs-computing.com

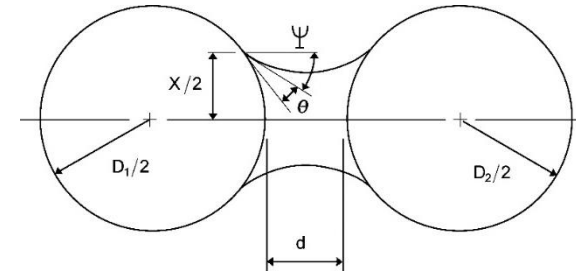
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Liquid Bridge Models

- Particle liquid on leads to
 - (a) liquid bridge force [capillary+viscous],
 - (b) liquid transfer
- Particles could be assumed to be solid or porous



	Washino Kimiaki	Easo
Liquid properties required	<ul style="list-style-type: none"> • Surface tension • Contact angle • Viscosity • Minimum separation distance ratio for viscosity calculations (~1.0% of smallest particle size); prevents viscous force from becoming exceedingly large • Maximum separation distance ratio, needed because neighbor lists need a cut-off 	<ul style="list-style-type: none"> • Surface tension • Contact angle • Viscosity • Min separation distance • Maximum separation distance ratio, needed because neighbor lists need a cut-off
Volume of liquid involved	Total volume of liquid available between the two spheres multiplied by some user-specified factor (0.05) Currently this 0.05 is hard-coded, but could be easily implemented in a way that the user can specify via script	Shi & McCarthy (2009); contributed fraction depends on relative particle sizes; for equal sized spheres, each contributes approximately 0.067 of its individual liquid volume to the bridge volume
Capillary force	Approximate theoretical solution using minimum energy approach (Rabinovitch, 2005)	Semi-empirical solution to the Young-Laplace equation (Soulie et al, 2006)
Viscous force	Nase et al as quoted in Shi&McCarthy	Nase et al as quoted in Shi&McCarthy
Formation distance	Rupture distance (see below)	Contact distance
Rupture distance	Lian et al, 1993 $D = \left(1 + \frac{\theta}{2}\right) V^{1/3}$ Not quite sure how theta_i / theta_j would transfer into a theta_effective in case two particles have different thetas, at first glance it seems that this is not covered in the paper. For now, I am assuming 0.5*(theta_i+theta_j)	Lian et al, 1993 $D = \left(1 + \frac{\theta}{2}\right) V^{1/3}$ Not quite sure how theta_i / theta_j would transfer into a theta_effective in case two particles have different thetas, at first glance it seems that this is not covered in the paper. For now, I am assuming 0.5*(theta_i+theta_j)
Liquid transfer between particles	Equal distribution method (Mani et al, 2013)	Equal distribution method (Mani et al, 2013)
Initialization	Specify default amount of liquid per particle in % of weight; can be overridden by set command based on region/and/or other criteria	Specify default amount of liquid per particle in % of weight; can be overridden by set command based on region/and/or other criteria

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Particle Breakage

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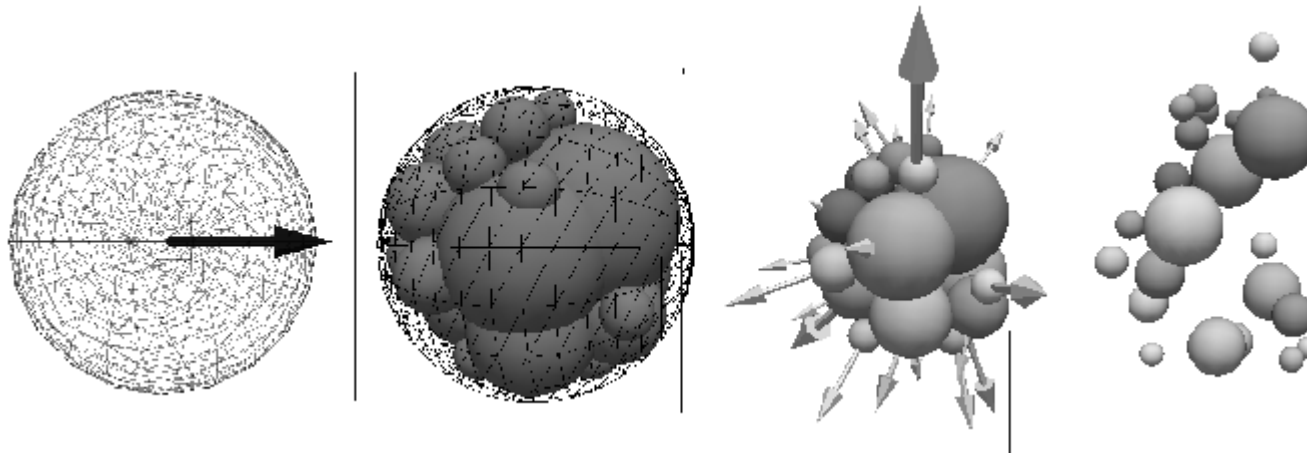
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Particle Breakage

- Particles can break when being processed – usually problem for process
- Breakage processes consume ~2-10% of worlds energy
- Spheres can be replaced by a conglomerate of daughter spheres



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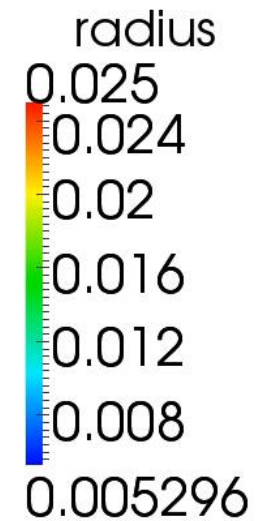
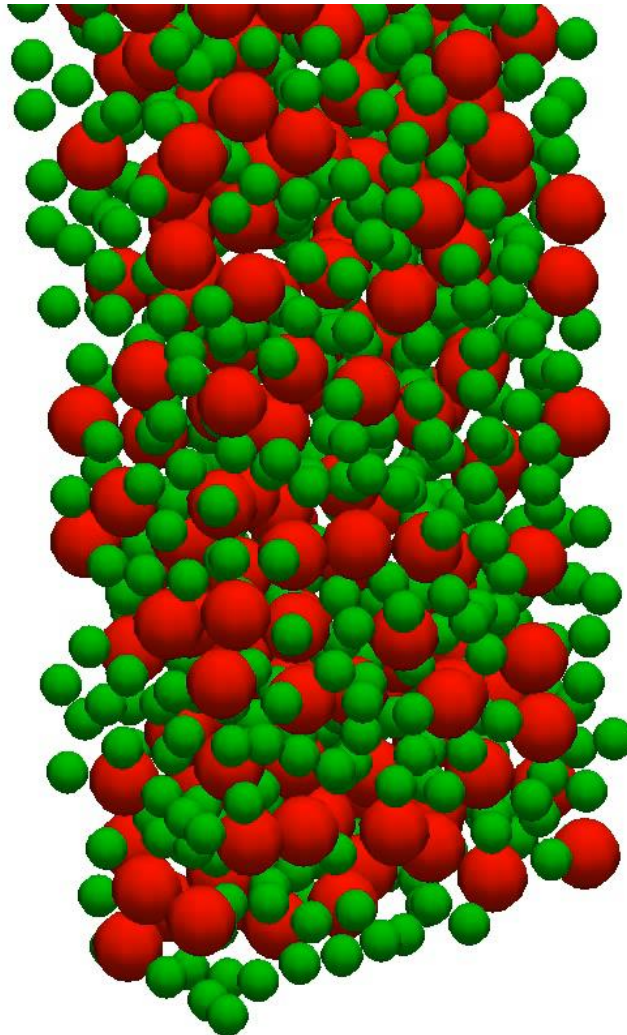
Simple Breakage Case

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Heat Conduction Coarse Grain

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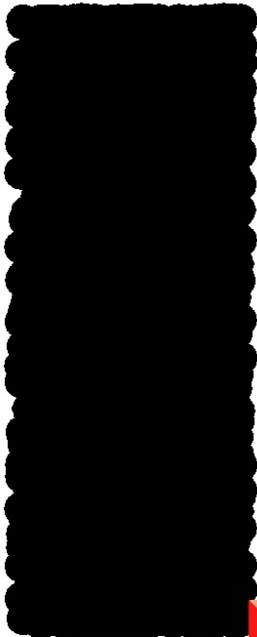
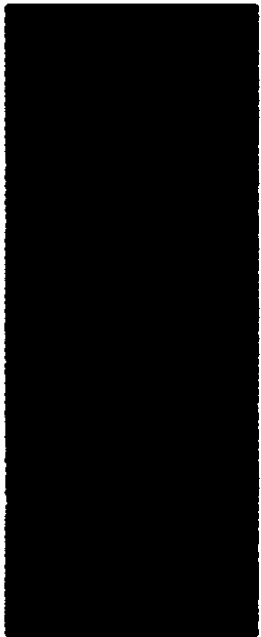
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Coarse Graining of Particle-Particle Heat Conduction

- Coarse graining = reproduce behaviour with larger particles
- Coarse graining was verified
- Temp profile over z after 20k time-steps, $dt = 1e-5s$ Fine (right) vs coarse (left) (2nd col = position, last col = averaged Temperature)



1	0.005	50	591.642	1	0.005	4073	594.263
2	0.015	66	581.198	2	0.015	4175	584.802
3	0.025	65	570.333	3	0.025	4159	574.599
4	0.035	62	558.195	4	0.035	4163	563.321
5	0.045	65	545.371	5	0.045	4141	550.846
6	0.055	57	530.978	6	0.055	4183	536.907
7	0.065	61	516.799	7	0.065	4153	521.661
8	0.075	65	500.365	8	0.075	4159	505.346
9	0.085	58	484.331	9	0.085	4154	488.244
10	0.095	67	467.624	10	0.095	4151	470.829
11	0.105	60	452.264	11	0.105	4127	453.845
12	0.115	64	438.969	12	0.115	4152	438.122
13	0.125	60	427.378	13	0.125	4147	424.841
14	0.135	66	417.807	14	0.135	4187	415.72
15	0.145	48	412.965	15	0.145	612	412.93

