

Garching, 29.01.2015

# **Interfacial Forces in Multi-Scale Two-Phase Flow**

## **OpenFOAM User Group Meeting, Garching**

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Supported by:

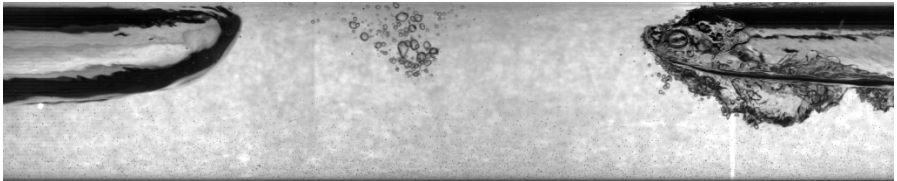


on the basis of a decision  
by the German Bundestag

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# Introduction

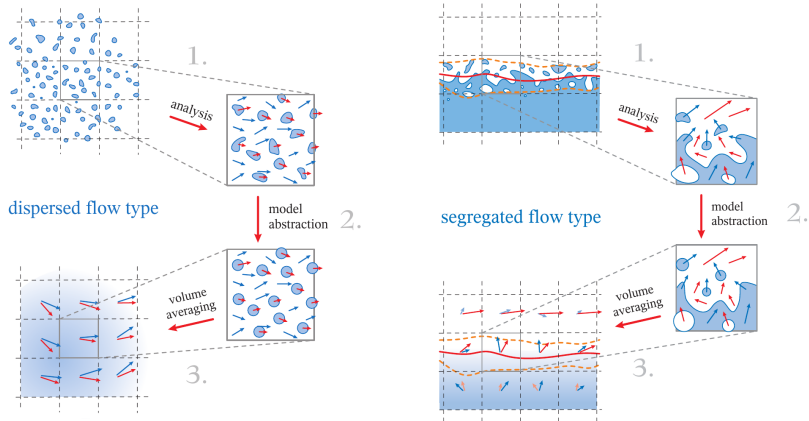
- two-phase flows problems often appear to be multi-scale problems
- to describe the flow, accurate calculation of interfacial mass and momentum transfer is crucial
- this is inherent feature of front tracking methods (VoF, Level-Set)
- must be achieved by additional closure in context of Two-Fluid Methodologies (TFM)



slug flow as multi-scale two-phase problem

# Partial Penetrating Continua

a TFM approach in multi-scale problems

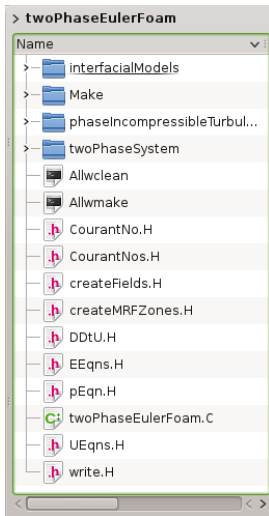


SOURCE: Marschall, H., 2011, *Towards the Numerical Simulation of Multi-Scale Two-Phase Flows*, PhD-Thesis, Technische Universität München, Germany

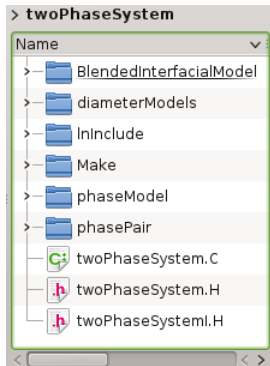
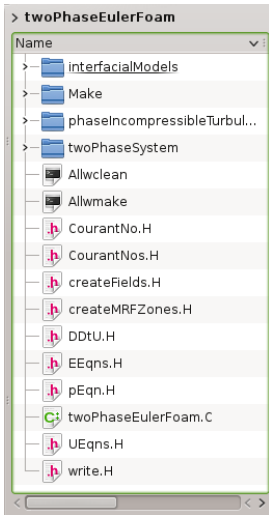
# Interfacial Forces in Multi-Scale Two-Phase Flow

$$\begin{aligned}
 \frac{\partial \alpha_k \rho_k \bar{\mathbf{U}}_k}{\partial t} + \nabla \cdot (\alpha_k \rho_k \bar{\mathbf{U}}_k \bar{\mathbf{U}}_k) - \nabla \cdot (\alpha_k \bar{\mathbf{R}}_k^{\text{eff}}) = & \\
 - \alpha_k \nabla \bar{p} + \alpha_k \rho_k \mathbf{g} & \text{ pressure and gravitation} \\
 + \Gamma_{fs} \cdot \sigma \kappa \nabla \alpha & \text{ surface Tension} \\
 + \Gamma_{fs} \cdot \alpha_l \alpha_g K^s (\bar{\mathbf{U}}_g - \bar{\mathbf{U}}_l) & \text{ drag - segregated flow} \\
 + (1 - \Gamma_{fs}) \cdot \alpha_l \alpha_g K^d (\bar{\mathbf{U}}_g - \bar{\mathbf{U}}_l) & \text{ drag - dispersed flow} \\
 + (1 - \Gamma_{fs}) \cdot \alpha_g \rho_l C_L (\bar{\mathbf{U}}_g - \bar{\mathbf{U}}_l) \times (\nabla \times \bar{\mathbf{U}}_l) & \text{ lateral Lift} \\
 + (1 - \Gamma_{fs}) \cdot \rho_l k_l C_{td} \nabla \alpha & \text{ turbulent dirpersion} \\
 + (1 - \Gamma_{fs}) \cdot \alpha_g \rho_l \left( \frac{D \bar{\mathbf{U}}_g}{Dt} - \frac{D \bar{\mathbf{U}}_l}{Dt} \right) & \text{ virtual mass} \\
 + (1 - \Gamma_{fs}) \cdot \rho_l \alpha_g C_W |\bar{\mathbf{U}}_g - \bar{\mathbf{U}}_l|^2 \vec{n}_W & \text{ wall lubrication}
 \end{aligned}$$

# Calculation of Interfacial Forces in OpenFOAM



# Calculation of Interfacial Forces in OpenFOAM



## Solution Procedure

executable twoPhaseEulerFoam calls twoPhaseEulerFoam.C:

```
Info<< "\ nStarting time loop \ n" << endl;
while (runTime.run())
{
    runTime++;
    Info<< "Time=" << runTime.timeName() << nl << endl;
    // --- Pressure-velocity PIMPLE corrector loop
    while (pimple.loop())
    {
        fluid.solve();
        rho = fluid.rho();
        fluid.correct();

        #include "EEqns.H"
        #include "UEqns.H"

        // --- Pressure corrector loop
        while (pimple.correct())
        {
            #include "pEqn.H"
        }
        if (pimple.turbCorr())
        {
            fluid.correctTurbulence();
        }
    }
    #include "write.H"
}
Info<< "End\ n" << endl;
return 0;
```

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        {
            fluid.correctTurbulence();
        }
    }
    #include "write.H"
}
Info<< "End\ n" << endl;
return 0;
```



## Solution Procedure

the "main" function `solve()` takes place in `twoPhaseSystem.C`:

```

void Foam::coupledTwoPhaseSystem::solve() {
// 1. DEFINE FLUXES:

    surfaceScalarField phic(" phic", phi_);
    surfaceScalarField phir(" phir", phi1 - phi2);

// 2. ADDED INTERFACE COMPRESSION TO FLUX:

    volScalarField GammaFs = findInterface();
    phir += min(GammaFs*mag(phic/mesh_.magSf()), max(phic/mesh_.magSf()))* - nHat_;

// 3. CALCULATE FLUXES:

    surfaceScalarField alphaPhic1
    (
        fvc::flux
        (
            phic,
            alpha1,
            alphaScheme
        )
        + fvc::flux
        (
            -fvc::flux(-phir, scalar(1) - alpha1, alpharScheme),
            alpha1,
            alpharScheme
        )
    );

```

...

```
...  
// 4. LIMIT PHASE FRACTIONS:  
    MULES::explicitSolve  
  
// 5. CALCULATE INTERFACE CURVATURE:  
    calculateCurv ();  
}
```

The interfacial forces are also calculated within `twoPhaseSystem.C`:

```
Foam::tmp<Foam::volScalarField> Foam::coupledTwoPhaseSystem::dragCoeff() const {...}  
Foam::tmp<Foam::volScalarField> Foam::coupledTwoPhaseSystem::virtualMassCoeff() const {...}  
Foam::tmp<Foam::volVectorField> Foam::coupledTwoPhaseSystem::liftForce() const {...}  
Foam::coupledTwoPhaseSystem::wallLubricationForce() const {...}  
Foam::coupledTwoPhaseSystem::turbulentDispersionForce() const {...}  
Foam::tmp<Foam::surfaceScalarField> Foam::coupledTwoPhaseSystem::surfaceTension() {...}
```

# Interfacial Forces in Multi-Scale Two-Phase Flow

$$\begin{aligned}
 \frac{\partial \alpha_k \rho_k \bar{\mathbf{U}}_k}{\partial t} + \nabla \cdot (\alpha_k \rho_k \bar{\mathbf{U}}_k \bar{\mathbf{U}}_k) - \nabla \cdot (\alpha_k \bar{\mathbf{R}}_k^{\text{eff}}) = & \\
 - \alpha_k \nabla \bar{p} + \alpha_k \rho_k \mathbf{g} & \quad \text{pressure and gravitation} \\
 + \Gamma_{fs} \cdot \sigma \kappa \nabla \alpha & \quad \text{surface Tension} \\
 + \Gamma_{fs} \cdot \alpha_l \alpha_g K^s (\bar{\mathbf{U}}_g - \bar{\mathbf{U}}_l) & \quad \text{drag - segregated flow} \\
 + (1 - \Gamma_{fs}) \cdot \alpha_l \alpha_g K^d (\bar{\mathbf{U}}_g - \bar{\mathbf{U}}_l) & \quad \text{drag - dispersed flow} \\
 + (1 - \Gamma_{fs}) \cdot \alpha_g \rho_l C_L (\bar{\mathbf{U}}_g - \bar{\mathbf{U}}_l) \times (\nabla \times \bar{\mathbf{U}}_l) & \quad \text{lateral Lift} \\
 + (1 - \Gamma_{fs}) \cdot \rho_l k_l C_{td} \nabla \alpha & \quad \text{turbulent dirpersion} \\
 + (1 - \Gamma_{fs}) \cdot \alpha_g \rho_l \left( \frac{D \bar{\mathbf{U}}_g}{Dt} - \frac{D \bar{\mathbf{U}}_l}{Dt} \right) & \quad \text{virtual mass} \\
 + (1 - \Gamma_{fs}) \cdot \rho_l \alpha_g C_W |\bar{\mathbf{U}}_g - \bar{\mathbf{U}}_l|^2 \vec{n}_W & \quad \text{wall lubrication}
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 + (1 - \Gamma_{fs}) \cdot \rho_l k_l C_{td} \nabla \alpha & \text{ turbulent dispersion} \\
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 \end{aligned}$$

# Surface Tension

continuum surface force: surfaceVector  $\rightarrow$  volumeVector

$$\int_S \sigma \kappa \delta(x - x') dS \approx \sigma \kappa \nabla \alpha$$

The momentum equation in /twoPhaseEulerFoam/UEqns.H

```

U1Eqn =
(
    fvm::ddt(alpha1, U1)
  + fvm::div(alphaPhi1, U1)
  + phase1.turbulence().divDevReff(U1)
  ==
    fvc::interpolate(fluid.surfaceTension())
  ...
)

```

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  ...
)
```

calls the calculation in twoPhaseSystem.C:

```
Foam::tmp<Foam::surfaceScalarField> Foam::coupledTwoPhaseSystem::surfaceTension()
{
    *fvc::interpolate(GammaFs)
    *sigma()
    *fvc::interpolate(Curv_)
    *(
        fvc::interpolate(phase2_)*fvc::snGrad(phase1_)
      - fvc::interpolate(phase1_)*fvc::snGrad(phase2_)
    )
}
```

## Used Geometric Functions

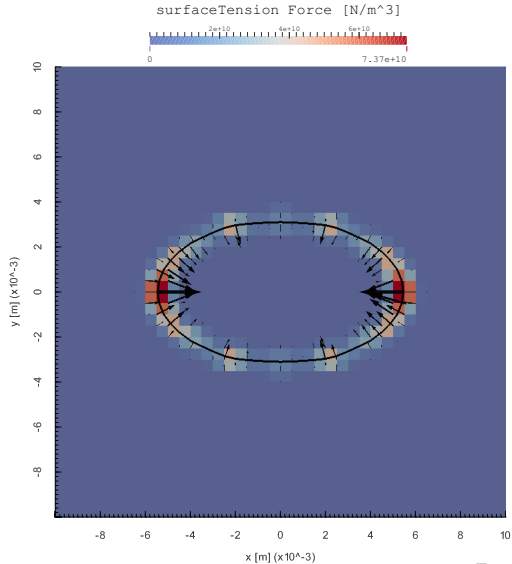
`fv::interpolate()` converts `volumeField<Type>` to `surfaceField<Type>` by central differencing

`fv::reconstruct()` converts `surfaceField<Type>` to `volumeField<Type>`

`fv::snGrad()` surface normal gradient  $\rightarrow$  returns `surfaceField<Type>`

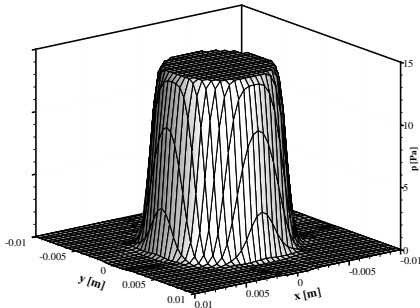
$$(\nabla\phi)_f = \frac{\phi_N - \phi_P}{|\vec{d}|}, \quad \vec{d} = N \rightarrow P$$

# Calculation of Surface Tension





# Pressure due to Surface Tension



pressure jump over bubble interface

- Laplace's law for infinite cylinder:

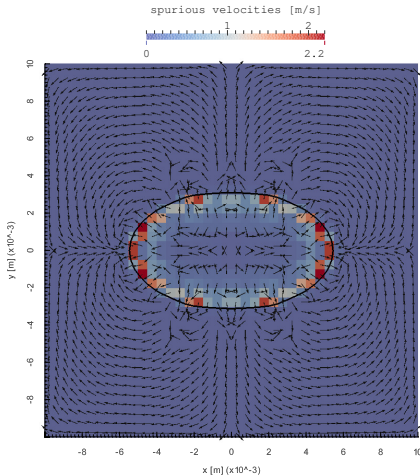
$$p_{bub} = \frac{\sigma}{r_{bub}},$$

gives analytical solution of ?

$p_{bub} = 14$  Pa.

- graphic shows the solution for 40x40 grid
- pressure smeared over several cell

# Spurious Velocities due to Surface Tension



- inaccuracy in curvature calculations leads to imbalance in forces
- in consequence, spurious velocities appear and disturb the interface ?
- to overcome this problem:
  - geometrical reconstruction of the interface
  - mesh adaptive methods<sup>1</sup>

1 Ganesan, S., 2008, *On spurious velocities in problems with incompressible interfaces flow*, Otto-von-Guericke-Universität Magdeburg, Germany

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# Drag Calculation

- also referred to as *interface momentum transfer term*
  - commonly based on the assumption of spherical particles, drag force for dispersed flow  $\vec{F}_{D,k}^d$  reads (OF standart):
- 
- in OpenFOAM  $\tau_{ij}$  is extracted from the equation
  - several common models to calculate drag coefficient  $C_D$  are available in the official release



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$$\vec{F}_{D,k}^d = \frac{3}{4}\alpha(1 - \alpha)\rho_l \frac{C_D}{d_p} |\mathbf{u}_g - \mathbf{u}_l| (\mathbf{u}_g - \mathbf{u}_l)$$

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- in OpenFOAM  $K = \frac{3}{4} \rho_l \frac{C_D}{d_P} |\mathbf{U}_g - \mathbf{U}_l|$  is extracted from the equation
- several common models to calculate drag coefficient  $C_D$  are available in the official release



# Drag Calculation

The momentum equation in /twoPhaseEulerFoam/UEqns.H

```
U1Eqn =  
(  
    fvm::ddt(alpha1, U1)  
+ fvm::div(alphaPhi1, U1)  
+ phase1.turbulence().divDevReff(U1)  
==  
    fvc::interpolate(fluid.surfaceTension())  
- fvm::Sp(fluid.dragCoeff()/rho1, U1)  
...  
)
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```
Foam::tmp<Foam::volScalarField> Foam::coupledTwoPhaseSystem::dragCoeff() const
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    phasePair::dictTable dragTable(lookup("drag"));

    const dragModel& dm = dragTable;

    volScalarField* Kptr_ =
    (
        dm.phase1()*dm.phase2()
        *dm.K(mag(dm.phase1().U() - dm.phase2().U()))
    ).ptr();

    dragCoeff->Kptr_;

    return dragCoeff;
}
```

?



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```

$$\Rightarrow \alpha_l \alpha_g K (|\bar{\mathbf{U}}_g - \bar{\mathbf{U}}_l|)$$

?

```
return dragCoeff;
```

```
}
17/19
```

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    dragCoeff->Kptr_;
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```
return dragCoeff;
```

```
}
17/19
```

$\Rightarrow \alpha_l \alpha_g K (|\bar{\mathbf{U}}_g - \bar{\mathbf{U}}_l|)$  ?

models can be added modular!

# Summary

- several common sub-models in two-phase flow are available in the official release
- extending them or add new models can be achieved by acceptable effort
- modular composition of OpenFOAM makes it possible to modify submodels without changing the whole code
- physical weaknesses are sometimes accepted in favor of stability and numerical effort (e.g. interface compression)
- OpenFOAM shows up to be a *TOOLBOX* and should be used as that

**Thanks for your attention!**