

Integrated Development Environment (IDE) Eclipse for OpenFOAM®

Assessing the Performance of bubbleFoam

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1 Introduction

"Eclipse is an open source community whose projects are focused on building an extensible development platform, runtimes and application frameworks for building, deploying and managing software across the entire software lifecycle. Many people know us, and hopefully love us, as a Java IDE but Eclipse is much more than a Java IDE." - www.eclipse.org

Scope and objective of this tutorial is the introduction of the *Integrated Development Environment* (IDE) Eclipse for OpenFOAM®. Eclipse is a powerful IDE originally developed for Java programming. But with the *C/C++ Development Toolkit* (CDT) extension Eclipse becomes a very common IDE for *fast and efficient* C++ programming.

Due to the amount of advantages of using an IDE only a few of them are listed below.

- Well-arranged graphical user interface offering project explorer, outline, ...
- Fully integrated powerful text editor offering code highlighting, autocompletion, ...
- Integrated compiler offering linked error and warning marks
- Integrated debugger and debugging environment offering breakpoints and variable information
- Project management: bookmarks and tasks
- Extension: version management, multiple language support (Java, Python, ...)

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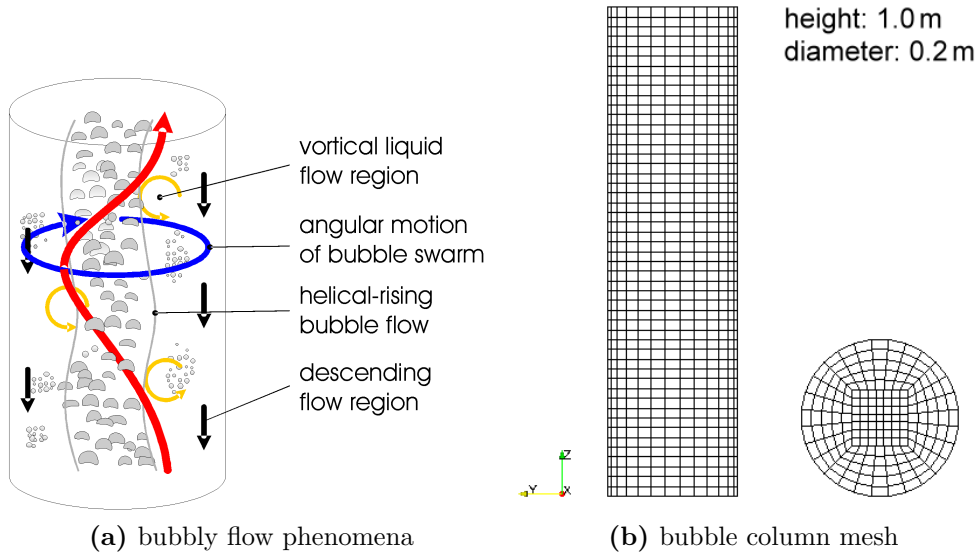


Figure 1: Bubbly flow in a bubble column

For further information concerning `Eclipse` refer to [1, 2, 3].

As `OpenFOAM` deals with physics we shall do so as well for the introduction of `Eclipse`: in this tutorial we will deal with bubbly flows (as they occur in bubble columns). In `OpenFOAM` the `bubbleFoam` solver enables to simulate the complex flow dynamics of these kind of gas-liquid flows. Currently, there is only the skeleton of a state-of-the-art model available and implemented in `bubbleFoam`: a two-fluid model framework based upon the Eulerian-Eulerian method. In this model, the flow morphology (i.e., the bubbles' shape) is not resolved explicitly at all, but is taken into account in an averaged manner presuming a specific shape. I.e., *conditional volume-averaging* of two-phase conservation equations results in the concept of interpenetrating continua [5], in which *all phase interactions* have to be modeled in order to physically close the system of governing equations. Thus, closure modeling is of major importance in order to gather reliable results. We will have a look at bubbly forces, which can be categorized further into drag and non-drag forces (\rightarrow slides). These forces essentially characterize the fluid dynamics of the two-phase flow system present in a bubble column.

The **motivation** of this tutorial rests in the fact that the already existing `bubbleFoam` solver needs to be restructured: i.e., we want to implement `runTime`-selective models for the calculation of interfacial forces. For this purpose, the existing 'hard-coded' models have to be rearranged in C++ libraries. Moreover, new models for both drag and non-drag forces have to be added. The rearrangement to libraries allows `runTime`-selective access to the interfacial force models during the simulation. The remainder of this tutorial will explain how this can be accomplished using `Eclipse`.

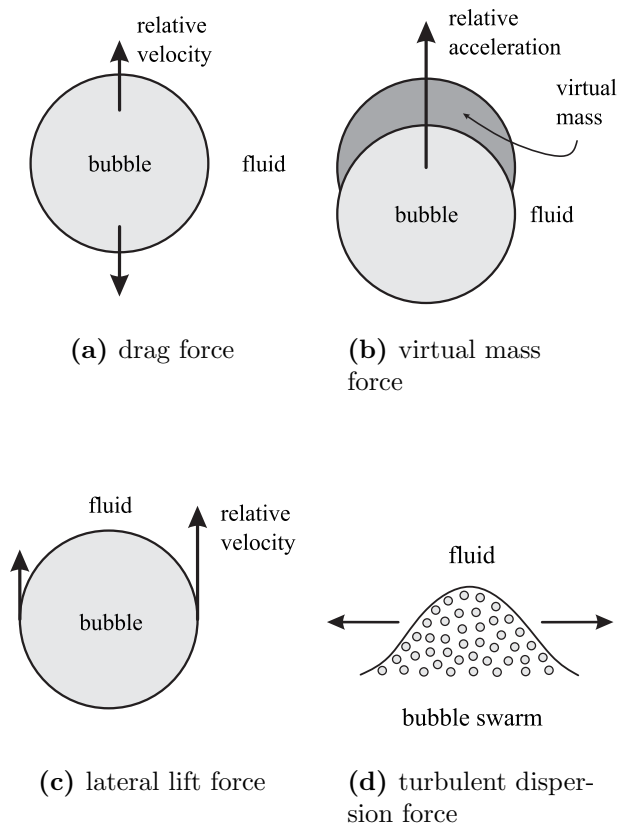


Figure 2: Bubble Forces – drag and non-drag forces [4]

2 Adding a runTime-selective Model – Drag Force

2.1 Preliminary steps

Start the OpenFOAM-1.6-ext-dbg terminal, as the debugging in the following tutorial will need the debug compiled binaries. Please export the path to the GNU Debugger (GDB) for Eclipse.

- `export PATH=/home/ubuntu/gdb4Eclipse/bin:$PATH`

Begin creating a personal version of bubbleFoam solver and copy the cylindricBubbleColumn test case in the correct `$WM_PROJECT_USER_DIR` directories.

- bubbleFoam solver:
`/usr/lib/OpenFOAM-1.6-ext-dbg/applications/solvers/multiphase/bubbleFoam`
- cylindricBubbleColumn test case:
`/cdrom/OFW6/Training/case-cylindricBubbleColumn.tgz.`

Change the name of the bubbleFoam folder into bubbleFoamMod and run `blockMesh` on the cylindricBubbleColumn.

2.2 Modifying bubbleFoam

1. Setting up Eclipse for OpenFOAM

- (a) Launch Eclipse using `eclipse &` and choose your workspace. If you're developing several projects it's advisable to create a workspace folders for each project. Herein we use the default workspace.
- (b) Change the developing environment to C++ in the menu bar under *Window → Show View → Other.. → C/C++ Projects*.
- (c) Make sure that *Project → Automatically build* in the menu bar is *unchecked*.
- (d) Import the bubbleFoamMod solver by creating a new C++ project in the menu bar under *Files → New → C++ Project*. Deactivate *Use default location*, then select the bubbleFoamMod folder. Set bubbleFoamMod as name for the project and click *Finish* (see Figure 3).
- (e) For using the OpenFOAM-specific compilation script `wmake`, change the compiler properties for Eclipse. Right-click on your new project in the project explorer on the left side, select *properties*. Setting the build command under *C/C++ Build*, deactivate the default build command and choose OpenFOAM's `wmake`. Deactivate *Generate Makefiles automatically*. Set the build directory - maybe you have to remove the `/Release` or `/Debug`. Confirm with *OK* (see Figure 4).

2. Developing and compiling with Eclipse

- (a) For the actual developing and compiling procedure, open your project folder in the project explorer window and double click a file, so the editor will open the file in the middle of your screen. You can now edit your file comfortably with the Eclipse text editor. Make line number visible by *Right-Click → Preferences.. → Editor*. Click on *Text Editors* and check *Show line numbers*.

- (b) For adapting your new solver `bubbleFoamMod` make the adequate changes in `Make/files` and rename `bubbleFoam.C` to `bubbleFoamMod.C` (see Figure 5). Moreover, place the executable at `EXE = $(FOAM_USER_APPBIN)/bubbleFoamMod`.
- (c) Recompile the solver by adding the new make targets `wmake` and `wclean`.
- First select the *Make Targets*-tab and select the directory where your `Make` folder is located. Create a new make target, by clicking on the *new make target* button. For compiling an application give the target a descriptive name – we choose `wmake` - leave the make target plain. The default build command is `wmake`.
 - Create the corresponding `wclean` – do not forget to change the build command to `wclean`.
 - Execute the *make targets* by double-clicking.

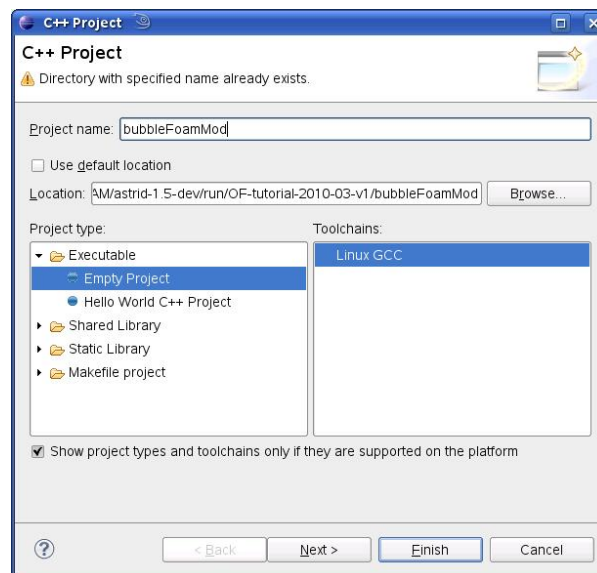


Figure 3: Import of `bubbleFoamMod`

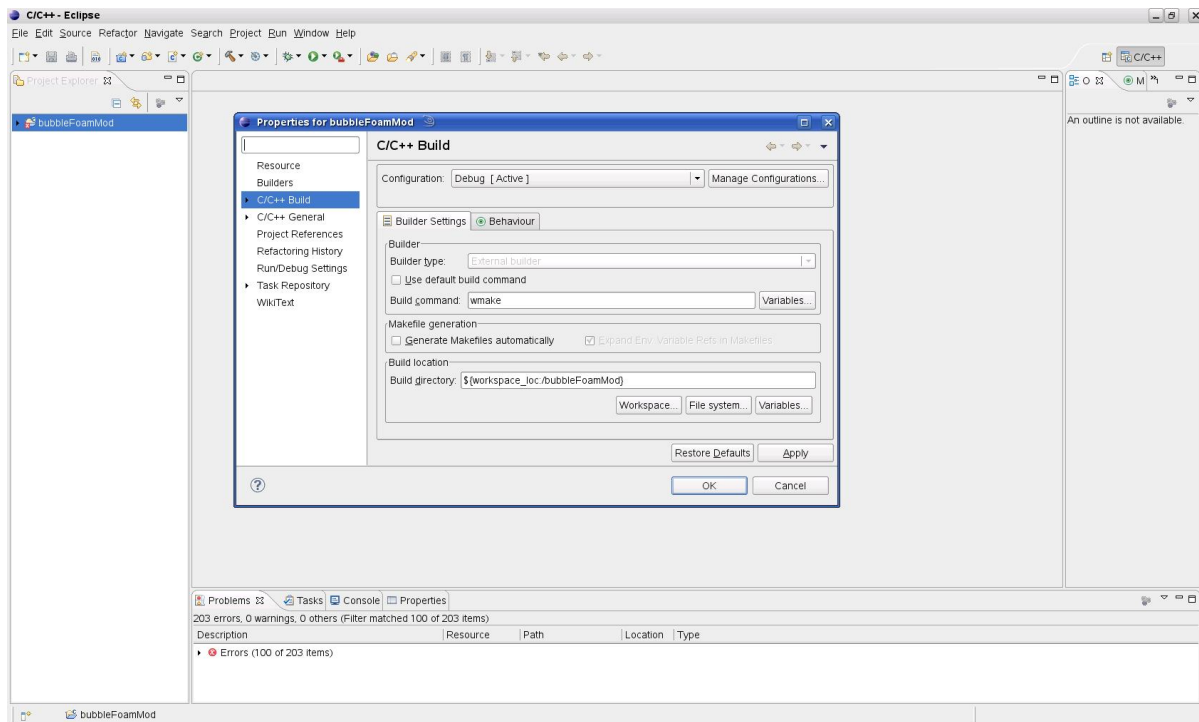


Figure 4: Eclipse Compiler Settings

```
dragModels/dragModel/dragModel.C
dragModels/dragModel/newDragModel.C
dragModels/SchillerNaumann/SchillerNaumann.C

LIB = $(FOAM_USER_LIBBIN)/libEulerianInterfacialModels
```

Source Code 1: interfacialModels/Make/files

```
EXE_INC = \
    -I$(LIB_SRC)/finiteVolume/lnInclude \
    -I../phaseModel/lnInclude

LIB_LIBS = \
    -L$(FOAM_USER_LIBBIN) -lphaseModel
```

Source Code 2: interfacialModels/Make/options

```
phaseModel/phaseModel.C

LIB = $(FOAM_USER_LIBBIN)/libphaseModel
```

Source Code 3: phaseModel/Make/files

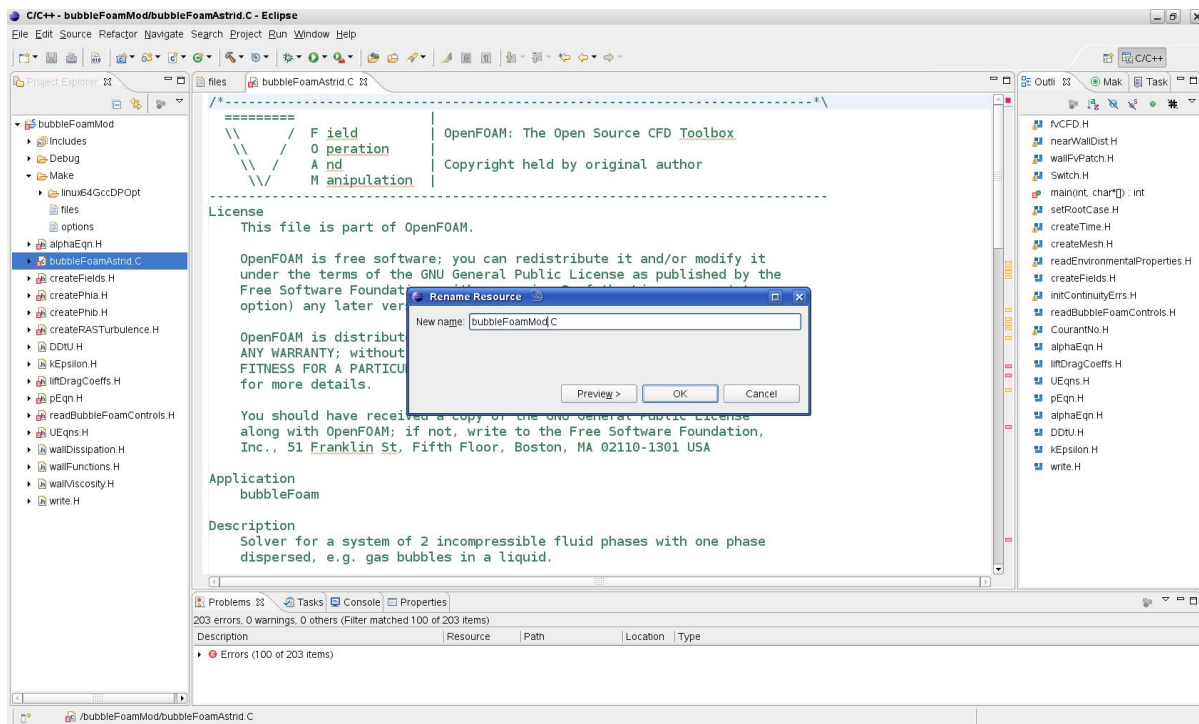


Figure 5: Eclipse Working Environment - Renaming bubbleFoamMod

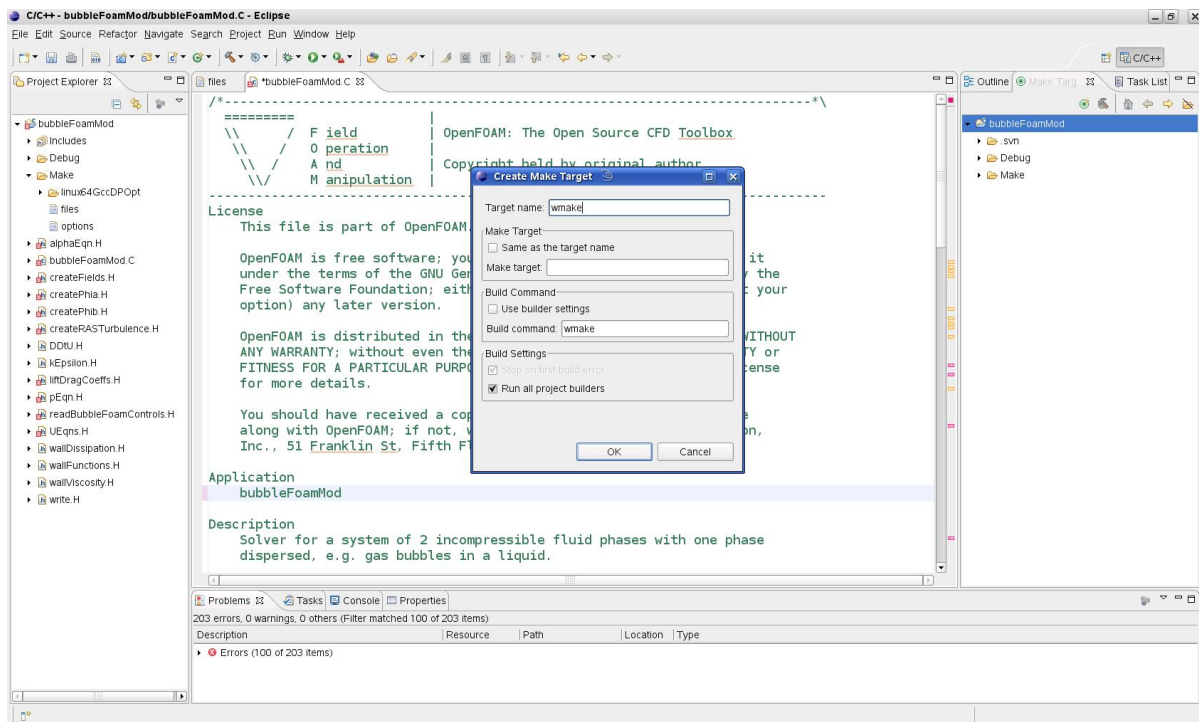


Figure 6: Create make targets

```
EXE_INC = \  
-I$(LIB_SRC)/finiteVolume/lnInclude \  
-I$(LIB_SRC)/transportModels/incompressible/lnInclude \  
-IinterfacialModels/lnInclude \  
-IphaseModel/lnInclude \  
-Iaveraging  
  
EXE_LIBS = \  
-lfiniteVolume \  
-lmeshTools \  
-lincompressibleTransportModels \  
-L$(FOAM_USER_LIBBIN) -lphaseModel \  
-L$(FOAM_USER_LIBBIN) -lEulerianInterfacialModels
```

Source Code 4: Make/options

3. Altering bubbleFoam

After the change of the name and recompiling the new solver, we will implement the drag and phase models as their own classes instead of the hard-coded version as currently implemented. Therefore, take some files from the `twoPhaseEulerFoam` solver, where some physical models for fluidized beds are already implemented dynamically.

- `twoPhaseEulerFoam` solver:
 `/usr/lib/OpenFOAM-1.6-ext-dbg/applications/solvers/multiphase/..`
 `../twoPhaseEulerFoam` with subfolders `interfacialModels` and `phaseModel`
- (a) Return to the terminal, copy the folders `interfacialModels/` and `phaseModel/` into the main `bubbleFoamMod` directory.
- (b) Return to Eclipse and refresh your workspace under *File* → *Refresh* (*F5*).
- (c) Delete all subdirectories in `interfacialModels/dragModels` except `dragModel/` and `SchillerNaumann/`.
- (d) In the `Make`-folder of `interfacialModels/` adapt the files `files` (deleting the unused drag models) and `options`. The files should look now as given in the Sources 1 and 2.
- (e) Change `phaseModel/Make/files` appropriately. Eventually the file should read as given in Source 3.
- (f) Next in the `Make`-directory of `bubbleFoam`, include the new models in the file `options`. The file should look as illustrated in Source 4.
- (g) For having access to the new classes, phase model pointers for each phase a and b have to be defined and initialized in `createFields.H` (Source 5). The models will be read in as an entry in the `transportProperties` dictionary of the case. Use the arrow operator `->` in order to call the functions for the phase properties as density, viscosity and diameter.
- (h) Next a new dictionary has to be created in order to make `runTime-selectivity` available for all interfacial force models we are going to add. Therefore define the new dictionary `interfacialProperties` in `createFields.H` (Source 6) and implement one drag model pointer for each phase a and b.
- (i) Include the `dragModel.H` file in `bubbleFoamMod.C` as shown in Source 7, so it is available for the compiler and then for our new `runTime-selective` application.
- (j) Finally, alter the `liftDragCoeffs.H` by inserting the correct calculation of the drag coefficient using the new drag models according to Source 8.
- (k) For the compilation in Eclipse create new make targets for the phase and interfacial force models, respectively.
 - Click onto the `interfacialModels` in the *make targets* tab. In order to compile them as libraries – in OpenFOAM this would be done by typing `wmake libso` – name your target `libso` and don't change the default builder settings `wmake`.
 - In the same way create `wclean` targets for your classes. Finally your make targets should look as illustrated in Figure 8.
 - First compile the libraries and then compile the complete solver again. Do not forget to save all the files you have changed before.

At this point you may take advantage of one of **Eclipse**' powerful features - the fully linked error and warning marks. Selecting the *Problems* tab on the bottom lists all errors and warnings. Clicking on them opens the appropriate files where the error or warning is marked on the left margin.

```
Info<< "Reading transportProperties\n" << endl;

IOdictionary transportProperties
(
    IOobject
    (
        "transportProperties",
        runTime.constant(),
        mesh,
        IOobject::MUST_READ,
        IOobject::NO_WRITE
    )
);

autoPtr<phaseModel> phasea = phaseModel::New
(
    mesh,
    transportProperties,
    "a"
);

autoPtr<phaseModel> phaseb = phaseModel::New
(
    mesh,
    transportProperties,
    "b"
);

const dimensionedScalar& rhoa = phasea->rho();
const dimensionedScalar& nua = phasea->nu();
const dimensionedScalar& da = phasea->d();

const dimensionedScalar& rhob = phaseb->rho();
const dimensionedScalar& nub = phaseb->nu();
const dimensionedScalar& db = phaseb->d();

dimensionedScalar Cvm
(
    transportProperties.lookup("Cvm")
);

dimensionedScalar Cl
(
    transportProperties.lookup("Cl")
);

dimensionedScalar Ct
(
    transportProperties.lookup("Ct")
);
```

```
IOdictionary interfacialProperties
(
    IOobject
    (
        "interfacialProperties",
        runTime.constant(),
        mesh,
        IOobject::MUST_READ,
        IOobject::NO_WRITE
    )
);

autoPtr<dragModel> draga = dragModel::New
(
    interfacialProperties,
    alpha,
    phasea,
    phaseb
);

autoPtr<dragModel> dragb = dragModel::New
(
    interfacialProperties,
    beta,
    phaseb,
    phasea
);
```

```

/*-----*
=====
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```

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```

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Application
    bubbleFoam

Description
    Solver for a system of 2 incompressible fluid phases with one phase
    dispersed, e.g. gas bubbles in a liquid.

/*-----*/

#include "fvCFD.H"
#include "nearWallDist.H"
#include "wallFvPatch.H"
#include "Switch.H"
#include "dragModel.H"

// ***** //

```

Source Code 7: bubbleFoamMod.C

```

volVectorField Ur = Ua - Ub;
volScalarField magUr = mag(Ur);

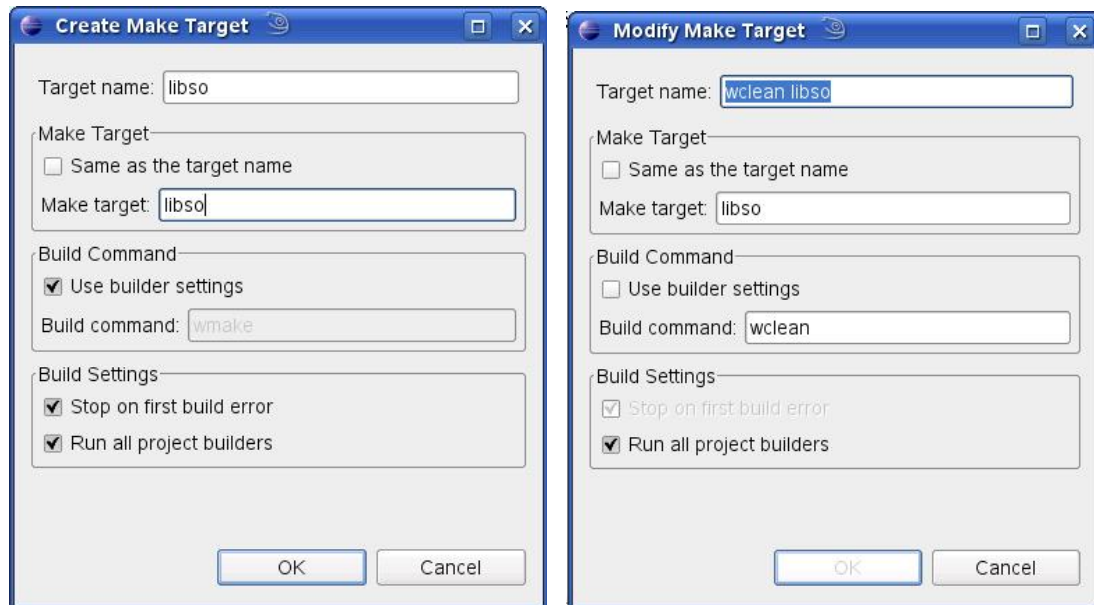
volScalarField Cda = draga->K(magUr);
volScalarField Cdb = dragb->K(magUr);

// corresponds to dragPhase == "blended" in twoPhaseEulerFoam
volScalarField dragCoef =
(
    "Cd",
    beta*Cda + alpha*Cdb
);

volVectorField liftCoeff = alpha*rhob*Cl*(Ur ^ fvc::curl(Ub));

```

Source Code 8: liftDragCoeffs.H



(a) C++ library make target

(b) C++ library clean target

Figure 7: Eclipse make target for C++ libraries

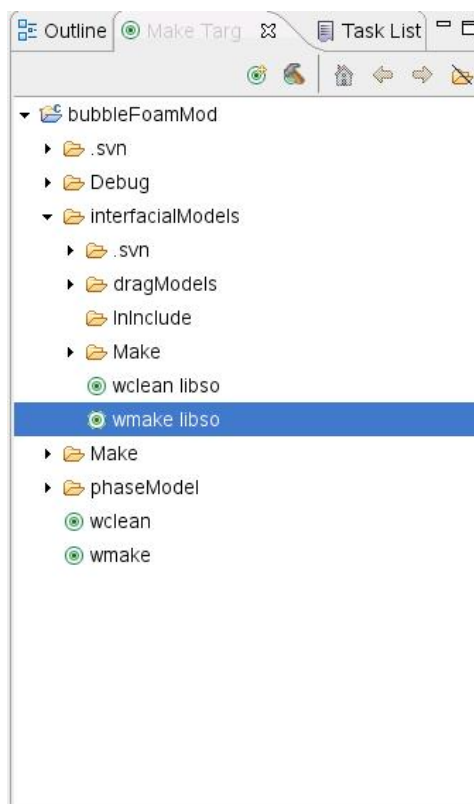


Figure 8: Overview: make targets

2.3 Implementing the models

After giving the solver the adequate C++ class base structure, new interfacial force models can be implemented easily.

1. To start with the implementation of the Tomiyama drag model create the new directory Tomiyama98 in `bubbleFoamMod/interfacialModels/dragModels` and copy all files from `.../dragModels/SchillerNaumann` in the new subdirectory.
2. Rename (F2) to `Tomiyama98.C` and `Tomiyama98.H`, respectively.
3. Add the new model to the existing solver in the `bubbleFoamMod/interfacialModels/Make/files` file by adding the following line: `dragModels/Tomiyama98/Tomiyama98.C`.
4. For the implementation of a *new* drag force model, change the equations accordingly to the Tomiyama drag force model as given in Source 9 and 10, respectively. For the change of name you can use *Edit* → *Find/Replace*.
5. For the calculation of the Eötvös number the surface tension and gravity (as dimensionedScalars) are required. In order to make this properties available, some modifications to the phase model have to be made. Therefore read in the surface tension `sigma` and gravity `g` from a dictionary entry (see `phaseModel.C` – Source 11, lines 57 to 64). Add the lines 68 to 72 and lines 126 to 134 (access functions of `sigma` and `g`) in `phaseModel.H` as shown in Source 12.
6. Create make targets for the `phaseModel` library as done for the `interfacialModels` and compile it. Then recompile the `interfacialModels` library and at last the `bubbleFoamMod` solver.

```
/*-----*/
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\\      / M anipulation|
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/*-----*/

#include "Tomiyama98.H"
```

```

#include "addToRunTimeSelectionTable.H"

// * * * * * Static Data Members * * * * * //
namespace Foam
{
    defineTypeNameAndDebug(Tomiyama98, 0);

    addToRunTimeSelectionTable
    (
        dragModel,
        Tomiyama98,
        dictionary
    );
}

// * * * * * Constructors * * * * * //
Foam::Tomiyama98::Tomiyama98
(
    const dictionary& interfaceDict,
    const volScalarField& alpha,
    const phaseModel& phasea,
    const phaseModel& phaseb
)
:
    dragModel(interfaceDict, alpha, phasea, phaseb)
{}

// * * * * * Destructor * * * * * //
Foam::Tomiyama98::~Tomiyama98()
{}

// * * * * * Member Functions * * * * * //
Foam::tmp<Foam::volScalarField> Foam::Tomiyama98::K
(
    const volScalarField& Ur
) const
{
    volScalarField Re = max(Ur*phasea.d()/phaseb.nu(), scalar(1.0e-3));
    volScalarField Eo = (phaseb.rho()-phasea.rho())*phasea.g()*
        pow(phasea.d(),2)/phaseb.sigma()*Re/Re;

    // Tomiyama Correlation for contaminated systems
    volScalarField Cds = 24.*(scalar(1) + 0.15*pow(Re, 0.687))/Re;
    volScalarField Cdj = 8./3.*(Eo/(Eo+4.));

    forAll(Cds, celli)
    {
        if(Cds[celli] < Cdj[celli])
        {
            Cds[celli] = Cdj[celli];
        }
    }

    return 0.75*Cds*phaseb.rho()*Ur/phasea.d();
}

```

Source Code 9: Tomiyama98.C


```

    //- Construct from components
    Tomiyama98
    (
        const dictionary& interfaceDict,
        const volScalarField& alpha,
        const phaseModel& phasea,
        const phaseModel& phaseb
    );

    // Destructor
    ~Tomiyama98();

    // Member Functions
    tmp<volScalarField> K(const volScalarField& Ur) const;
};

} // End namespace Foam

#endif

```

Source Code 10: Tomiyama98.H

```

/*-----*
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/*-----*
// * * * * * Constructors * * * * * //
  g-
  (
    dict_.lookup("g")
  ),
  sigma_
  (

```

```
dict_.lookup("sigma")
),
```

Source Code 11: phaseModel.C

```
/*-----*/
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Class
  Foam::phaseModel

SourceFiles
  phaseModel.C

/*-----*/
/*-----*/
Class phaseModel Declaration
/*-----*/

  //- gravity
  dimensionedScalar g-;

  //- surface tension
  dimensionedScalar sigma-;
// Member Functions
  const dimensionedScalar& g() const
  {
    return g-;
  }

  const dimensionedScalar& sigma() const
  {
    return sigma-;
  }
}
```

Source Code 12: phaseModel.H

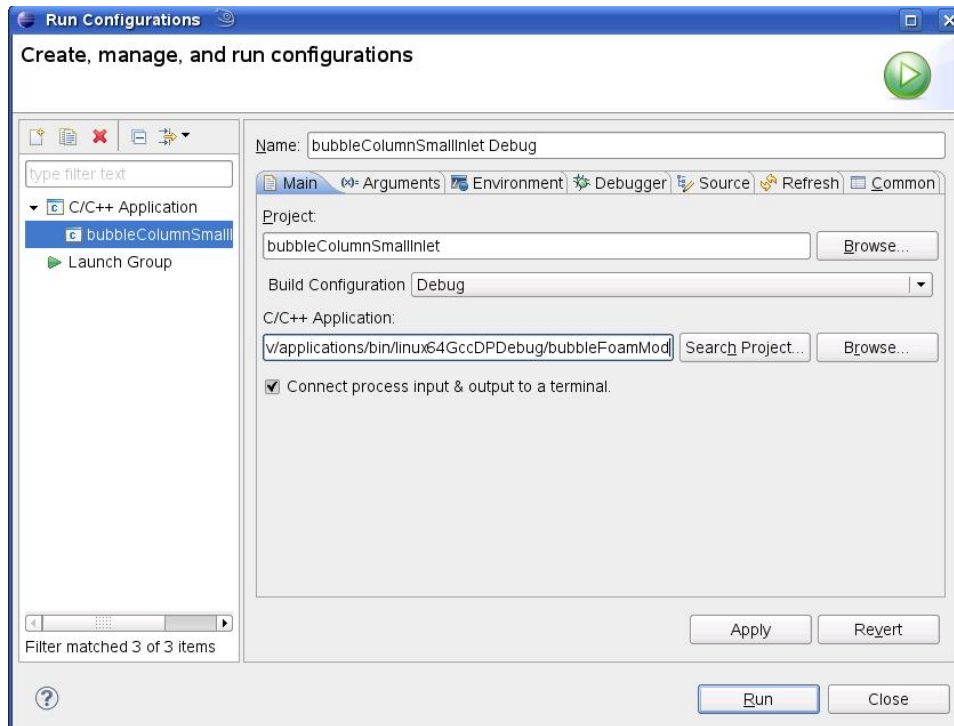


Figure 9: Run Configurations

3 Adjusting the case files

A simulation run can be accomplished within Eclipse, too.

1. Import the provided test case as a C++ project and adapt the project properties for OpenFOAM as mentioned in chapter 2.
2. The new drag force model can be selected via keyword entry in the dictionary `interfacialProperties`, while the new transport properties have to be added in the `transportProperties` dictionary.
3. Set up the run configurations in the menu bar under *Run* → *Run Configurations*. Double-click on *C/C++ Application*, choose your test case as project and your solver as C/C++ application. Here, the test case `cylindricBubbleColumn` is chosen as project and the solver `bubbleFoamMod` is chosen as application – be aware to select the correct binary file (see Figure 9).
4. Start your simulation by clicking on the Run button and have a look at the console output in Eclipse.

4 Debugging with Eclipse

One of Eclipse capabilities is efficient debugging. So, in this chapter we would like to check if the new model is implemented correctly and how the runTime-selective access to the models is carried out.

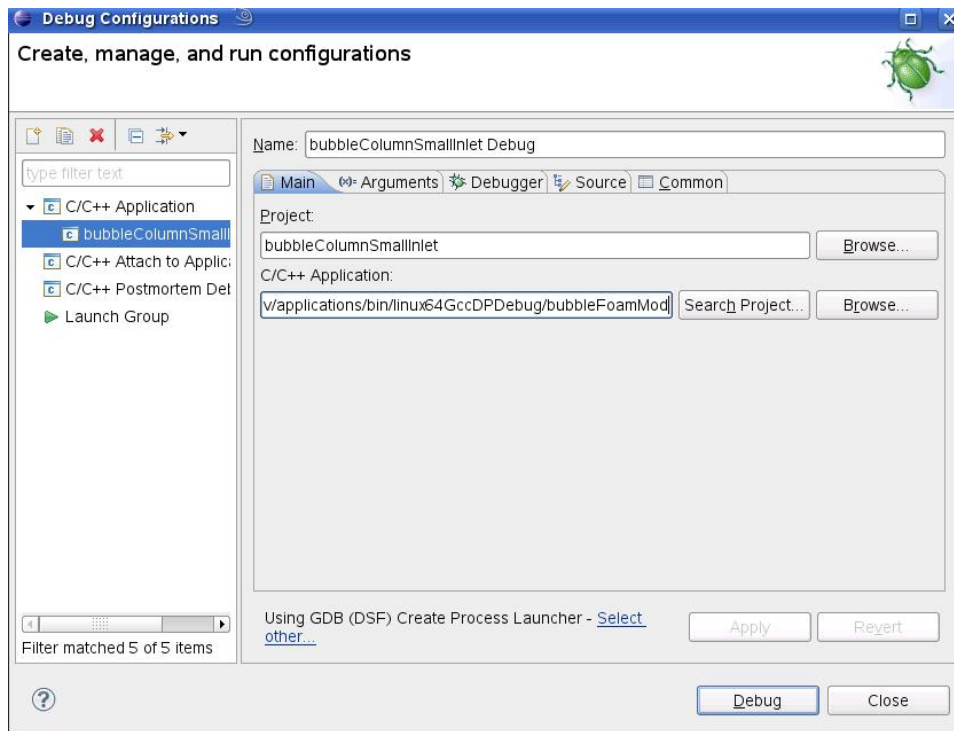


Figure 10: Debug Configurations

1. Set up the debugging configurations in *Run* → *Debug Configurations*. Choose the same settings for project and application as for the run configurations.
2. Make sure, that you are using the GDB (DSF) Create Process Launcher as debugger. If necessary, change it by clicking on *Select other...* (see Figure 10).
3. Start clicking on *Debug*. Now, the debug perspective should open - if not, activate it in the menu bar under *Window* → *Open Perspective* → *Debug*. The program should stop at the first breakpoint that is default set to entering `main{}`.

In Debug mode Eclipse allows you to set breakpoints by double-clicking on the bar to the left of the line numbers. You can resume debugging by clicking on the green play button (short cut *F8*). During debugging the console output on the bottom as well as the variable values and breakpoints on the right hand side are available (see Figure 11). You can walk through the code line-by-line while Eclipse highlights the line of the file you are currently computing. For stepping into function use the *step-into*-button (short cut *F5*) and for stepping over functions use the corresponding *step-over*-button (short cut *F6*).

4. Set the first breakpoint in `createFields.H` where you implemented the selection of the drag model (`autoPtr<dragModel> draga = dragModel::New()`). Set the second breakpoint in `liftDragCoeffs.H` where `Cds` is evaluated using the new drag model (`volScalarField Cda = draga->K(magUr)`).
5. Now resume to the first breakpoint (*F8*) and step into the drag model selection process (*F5*). The debugger now points to the constructor of the new drag model where the drag model to be chosen is read in from the dictionary entry in `interfacialProperties`.
6. Resume to the next breakpoint (*F8*) and step into (*F5*) the evaluation of the drag coefficient. Depending on the model chosen in the `interfacialProperties` dictionary

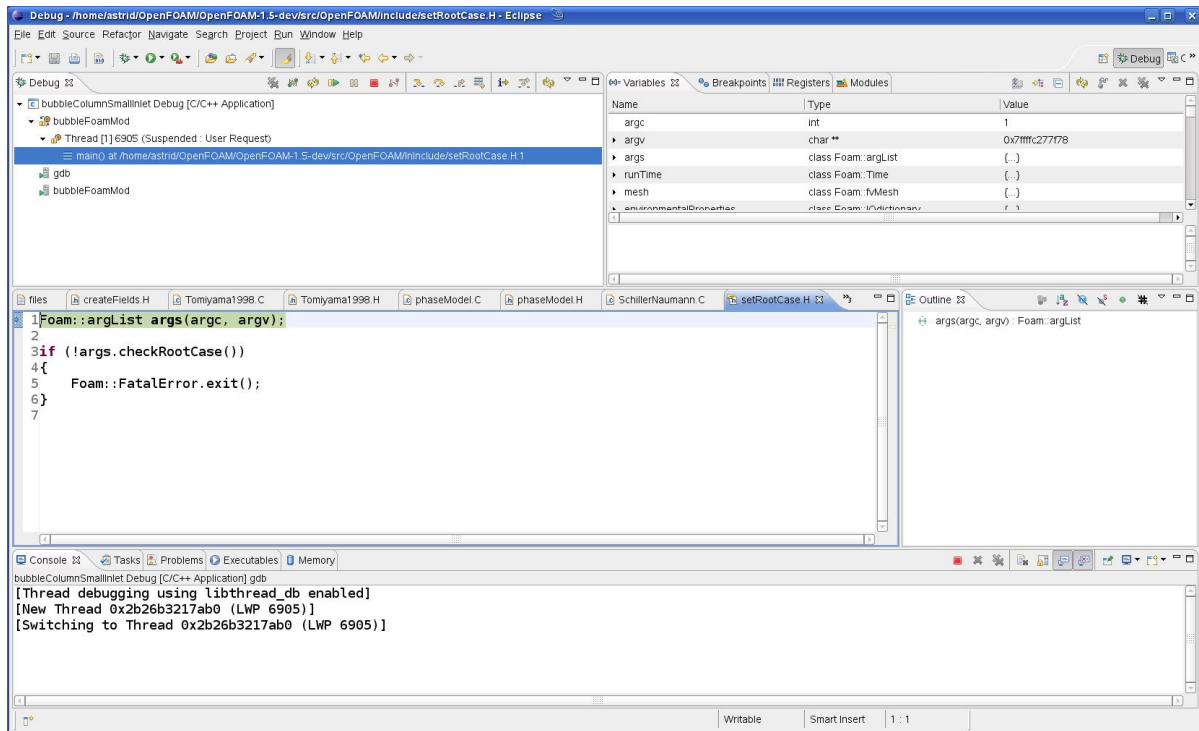


Figure 11: Debug Perspective

the debugger now points to either SchillerNaumann.C or Tomiyama98.C - for both cases calling the function `K(magUr)`. Step into (*F5*) the evaluation of the Reynolds number `volScalarField Re = max(Ur*phasea_.d()/phaseb_.nu(), scalar(1.0e-03));` and make sure that the bubble diameter of phase a (`phasea_.d()`) is accessed via access function of `phaseModel`. You can even check the value of the diameter variable in the *Variables* window on the right side.

A References

References

- [1] Cdt wiki. available online at <http://wiki.eclipse.org/index.php/CDT>, March 2010.
- [2] Eclipse homepage. available online at <http://www.eclipse.org>, March 2010.
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- [5] H. Rusche. *Computational fluid dynamics of dispersed two-phase flows at high phase fractions*. PhD thesis, Imperial College of Science, Technology and Medicine, London, 2002.

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